Name: ___________________

1.8b Lesson: The Objective Function

Vocabulary:

Objective Function = an equation for value of solution

Maximize/Minimize = the best answer

Example #1: A test is made up of multiple-choice and open-ended questions.

- It takes 3 minutes to do a multiple-choice question and 6 minutes for an open-ended question.
- You spend at least 60 minutes on the test
- You may answer no more than 20 questions.
- Multiple choice questions are worth 1 mark and open-ended questions are worth 3 marks

Define the relevant variables, write an inequality for each constraint in the question, draw the feasible region, and create an objective function that could maximize your score on the test.

m = multiple choice e = open-ended

$m, e \in W$

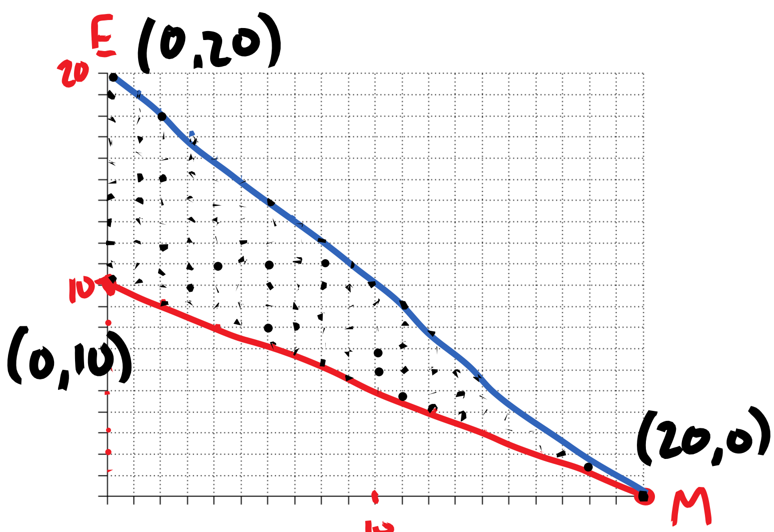
① $3m + 6e \geq 60$

$6e \geq -3m + 60$

$e \geq -\frac{1}{2}m + 10$

② $m + e \leq 20$

$e \leq -m + 20$



Objective Function Marks = 1m + 3e

$(0,10): 1(0) + 3(10) = 30$

$(0,20): 1(0) + 3(20) = 60$ MAX

$(20,0): 1(20) + 3(0) = 20$

Example #2: A refinery produces oil and gas and would like to optimize their revenue.

- At least 2L of gasoline is produced for each litre of heating oil
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for \$1.10 a litre and heating oil is projected to sell for \$1.75 a litre.

The company needs to determine the daily combination of gas and heating oil that must be produced in order to maximize revenue.

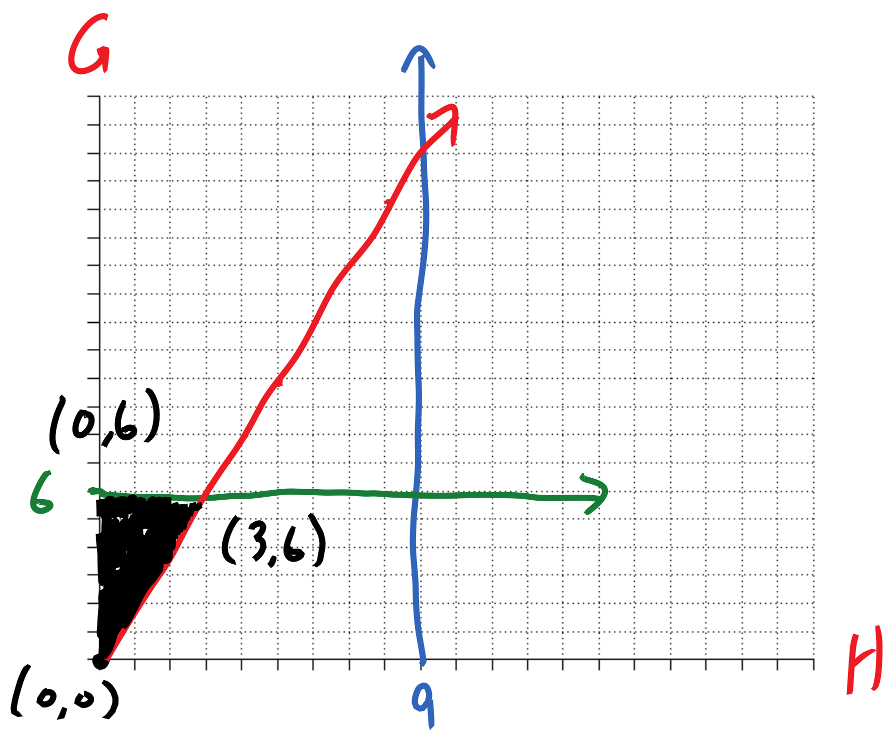
g: gasoline h: heating oil

$g, h \geq 0 \quad g, h \in \mathbb{R}$

① $g \geq 2h$

② $h \leq 9$

③ $g \leq 6$



Objective Function $R = \$1.10g + \$1.75h$

h, g		
$(0,0)$	$= \$1.10(0) + \$1.75(0) = 0$	
$(3,6)$	$= \$1.10(6) + \$1.75(3) = 11.85$	MAX
$(0,6)$	$= \$1.10(6) + \$1.75(0) = 6.6$	

1) Baskets of fruit are being prepared to sell. Maximize the number of fruit.

- Each basket contains at least 5 apples and at least 6 oranges
- Apples cost 20 cents each and oranges cost 35 cents each. The budget allows no more that $7, in total, for the fruit in each basket

a = apples g = oranges

$a, g \in W$

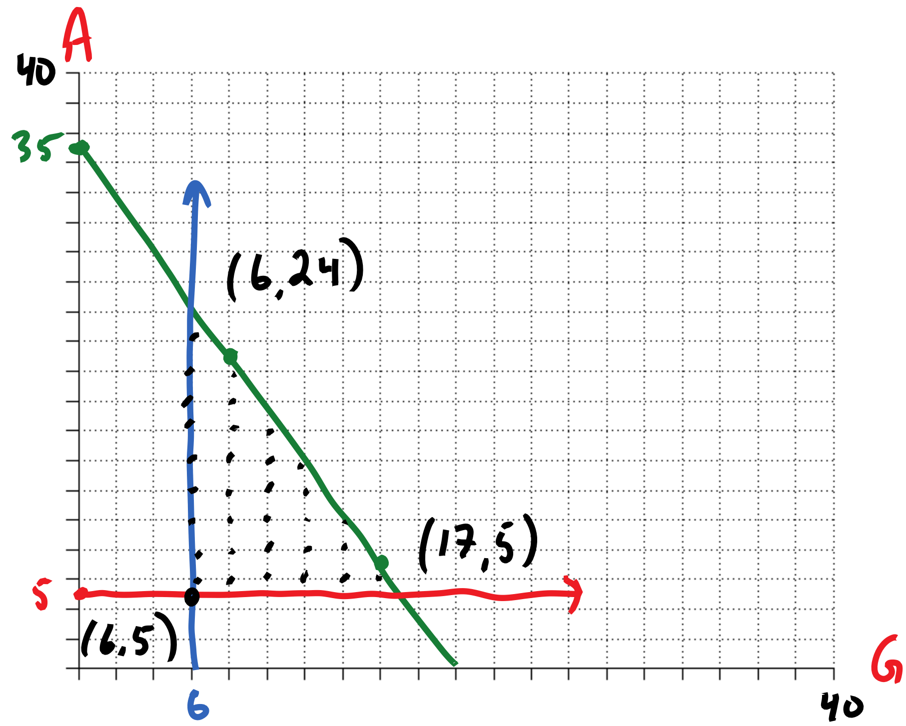
① $a \geq 5$

② $g \geq 6$

③ $.20a + .35g \leq 7$

$.20a \leq -.35g + 7$

$a \leq -\frac{7}{4}g + 35$



Objective Function # of Fruit = a + g

(6,5) : 6 + 5 = 11

(6,24) : 6 + 24 = 30 MAX

(17,5) : 17 + 5 = 22

2) A fast-food concession stand sells hotdogs and hamburgers.

- Daily sales can be as high as 300 hamburgers and hot dogs combined.
- The stand has room to stock no more that 200 hot dogs and no more than 150 burgers.
- Hot dogs cost $3.25, and hamburgers are sold for $4.75.

Create a model that could optimize the sales

h: hamburgers   d = hot dogs

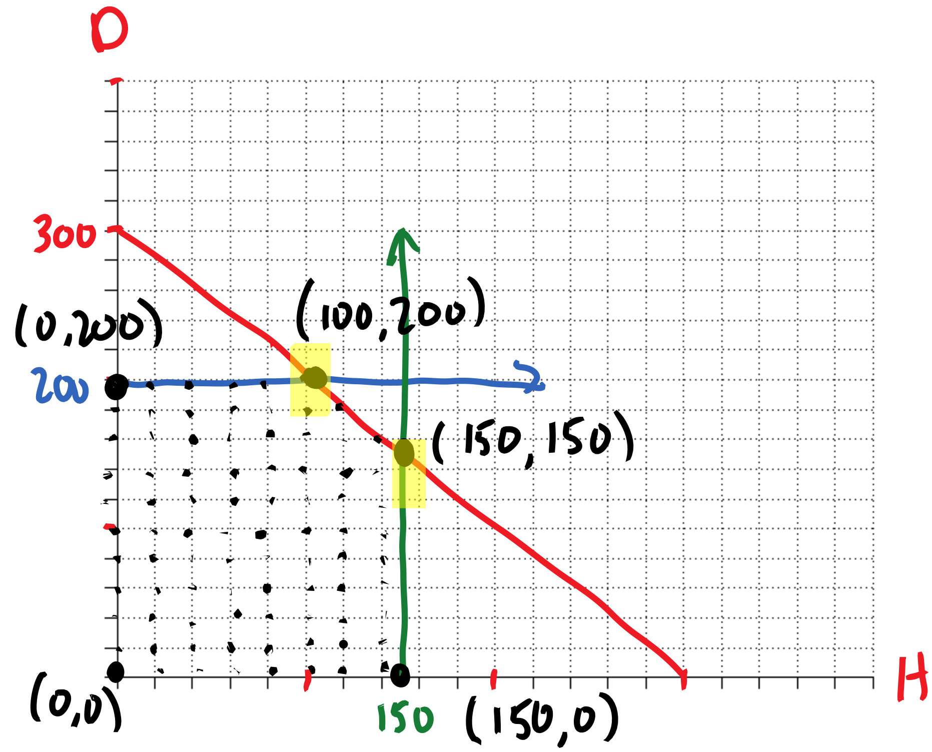
$h, d \in W$

① $h + d \leq 300$

$d \leq -h + 300$

② $d \leq 200$

③ $h \leq 150$



Objective Function Revenue = $4.75h + $3.25d

(0,0) =	= 0
(0,200) :	: $650
(150,0) :	: $712.50
(100,200) :	: $1125
(150,150) :	: $1200 MAX

3) A vending machine sells juice and pop.

- The machine holds, at most, 240 cans of drinks
- Sales from the vending machine show that at least 2 cans of juice are sold for each pop.
- Each can of juice sells for $1.00 and each can of pop sells for $1.25.

Maximize the revenue from this vending machine

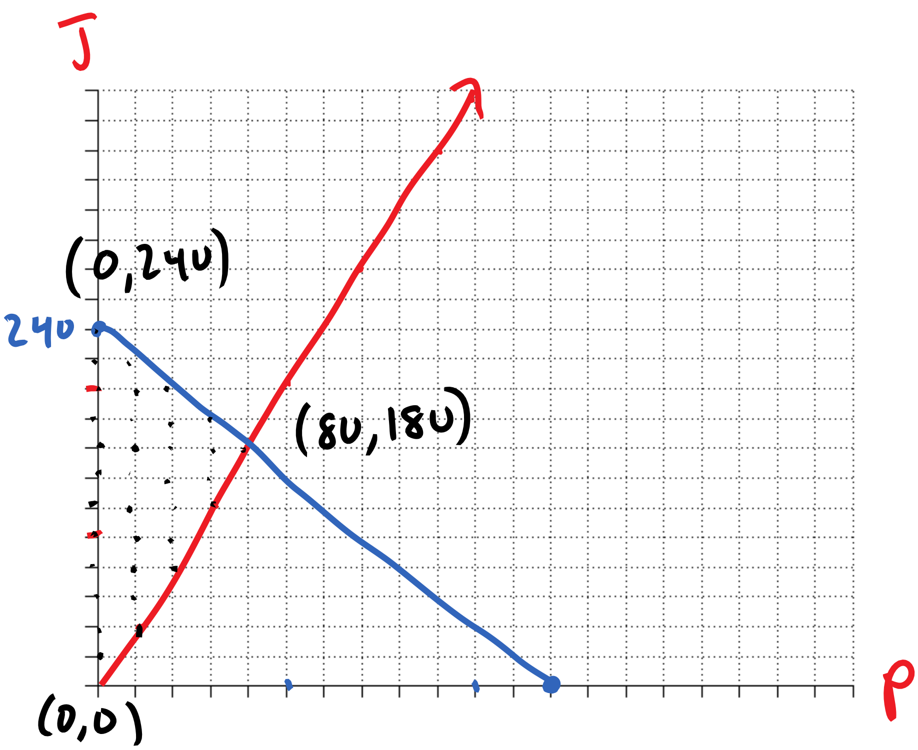
j = juice p = pop

$j, p \in W$

① $j + p \leq 240$

$j \leq -p + 240$

② $j \geq 2p$



Objective Function Revenue = $1.00 j + $1.25 p

p, j

(0,0) : = $0

(0,240) : = $240

(80,180) : = $280 MAX

4) A student council is ordering signs for the spring dance. Signs can be made in letter size or poster size.

- No more than 15 of each are wanted.
- At least 15 signs are needed altogether
- Letter-size signs cost \$9.80 each, and poster-size signs cost \$15.75 each

Minimize costs for the student council

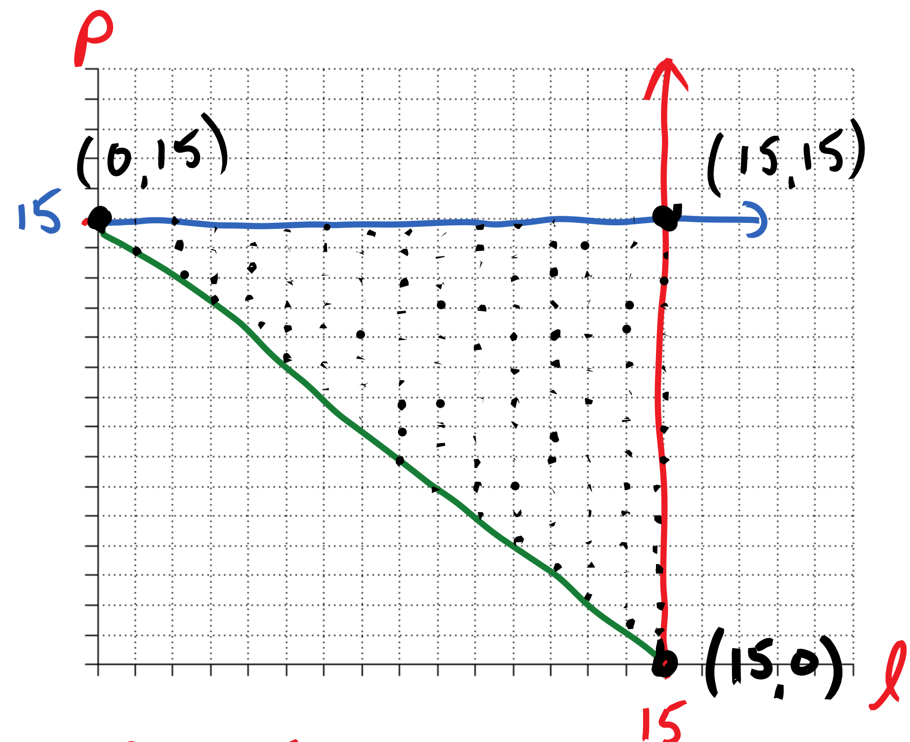
l = letter-size p = poster-size

$l, p \in W$

$l \le 15$

$p \le 15$

$l + p \ge 15$



Objective Function $Cost = 9.80l + 15.75p$

$(0,15) = 9.80(0) + 15.75(15) = 236.25$

$(15,15) = 9.80(15) + 15.75(15) = 383.25$

$(15,0) = 9.80(15) + 15.75(0) = 147.00$

MIN

5) A football stadium has 50 000 seats.

- Two-fifths of the seats are in the lower deck
- Three-fifths of the seats are in the upper deck
- At least 30 000 tickets are sold per game
- A lower deck ticket costs \$120, and an upper deck ticket costs \$80.

Maximize the revenue

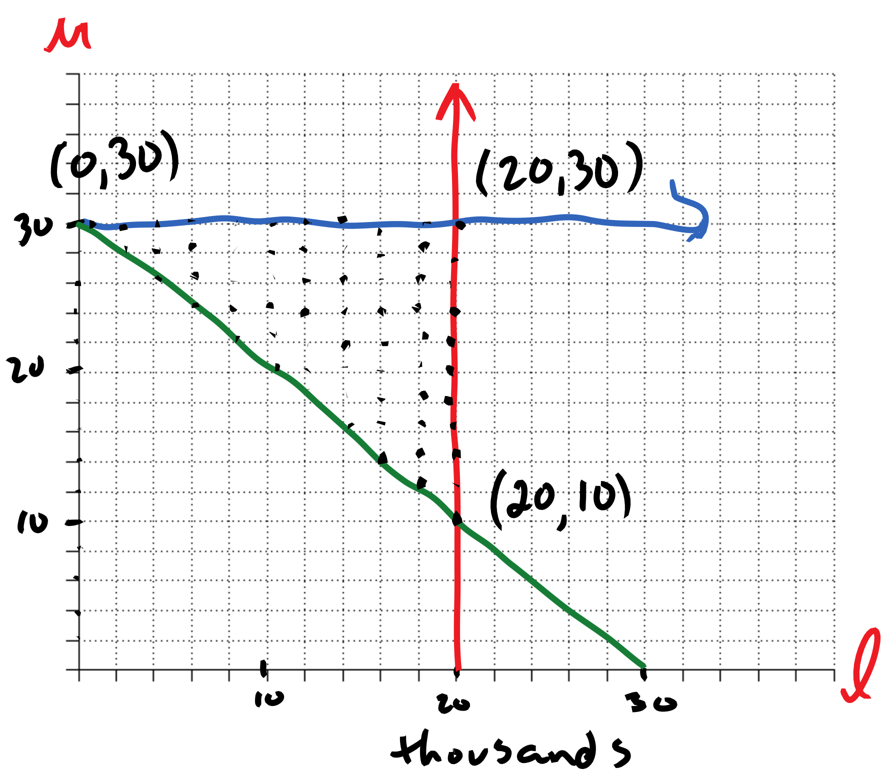
l : lower deck u = upper deck

$l, u \in W$

① $l \leq 20\,000$

② $u \leq 30\,000$

③ $l + u \geq 30\,000$



Objective Function Revenue = \$120l + \$80u

(0,30) : : \$2400

(20,30) : : \$4800 max

(20,10) : : \$3200

6) A Saskatchewan farmer is planting wheat and barley.

- He wants to plant no more than 1000 ha altogether
- The farmer wants at least three times as many hectares of wheat as barley
- The yield per hectare of wheat averages 50 bushels, and the yield per hectare of barley averages 38 bushels
- Wheat pays the farmer \$5.25 per bushel, and barley pays \$3.61 per bushel.

Maximize revenue for the farmer

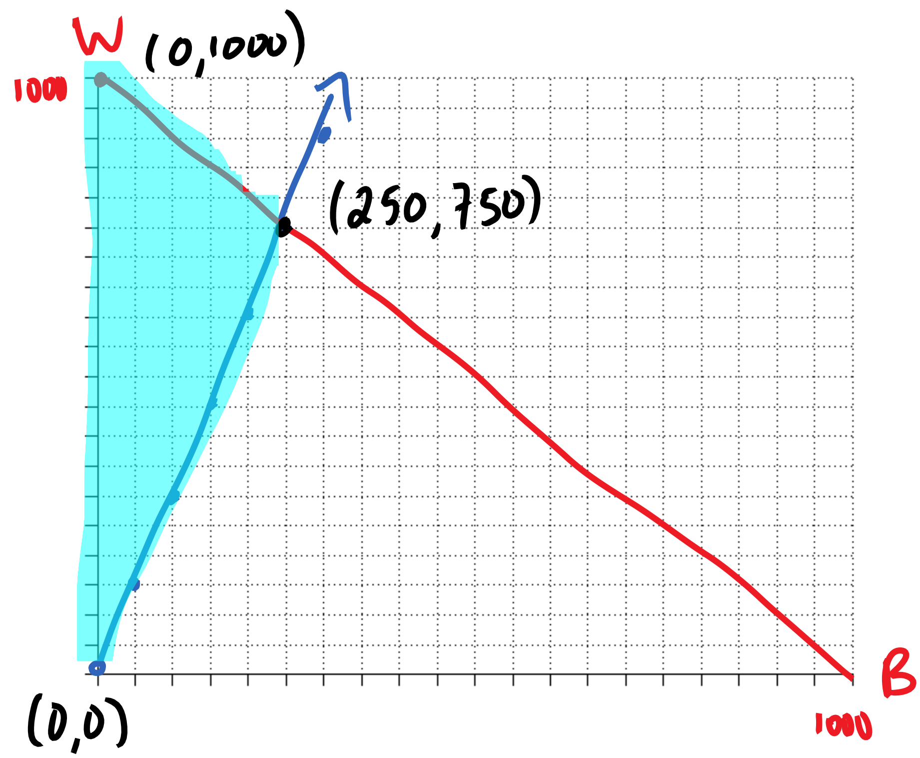
w: wheat b = barley

$w, b \in R$ $w, b \geq 0$

① $w + b \leq 1000$

$w \leq -b + 1000$

② $w \geq 3b$



Objective Function $R: 50(5.25)w + 38(3.61)b$ MAX

$(0, 1000): 262.5(1000) + 137.18(0) = 262500$

$(250, 750) = 262.5(750) + 137.18(250) = 231170$