

Chapter 3: Trigonometry**3.3 Sine Law**

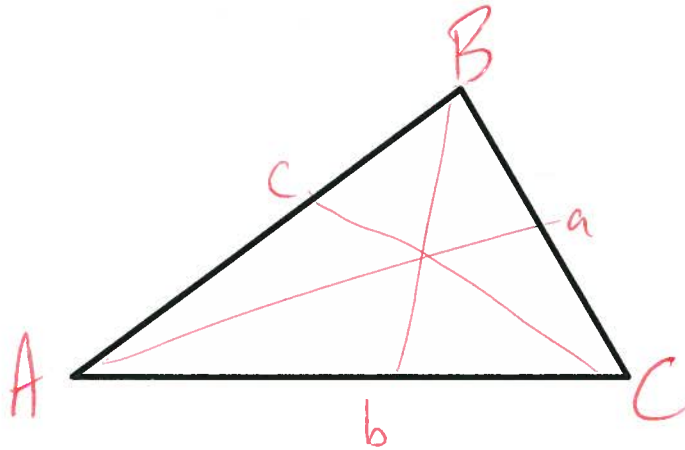
For solving some types of non-right triangles

The
Formula

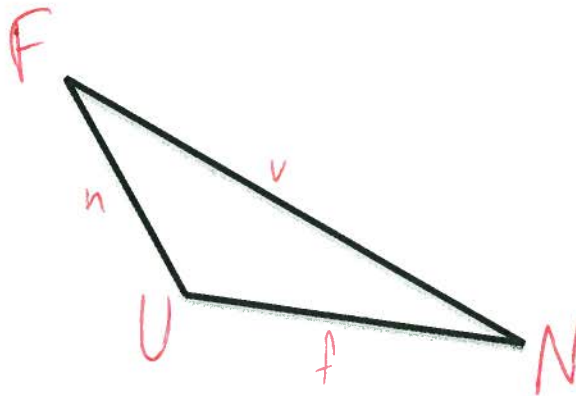
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Part 1 – Labeling a non-right triangle correctly

The Formula uses letters ABC like this:

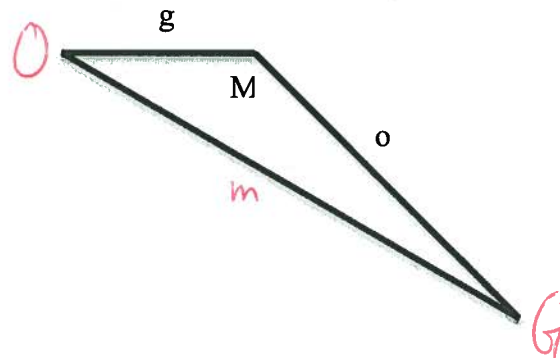
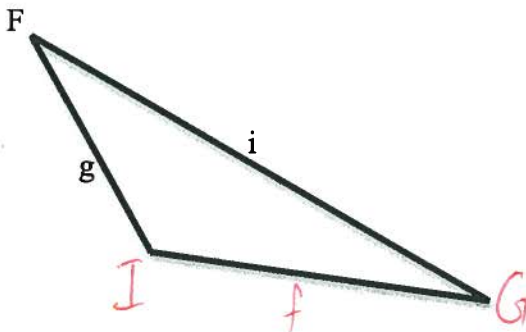
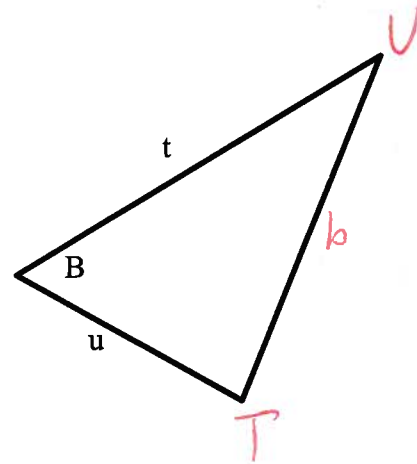
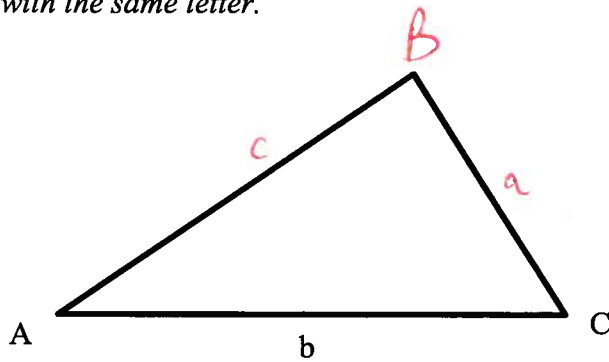


But any three letters can be used

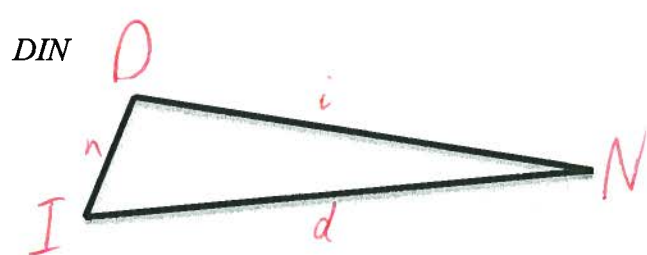
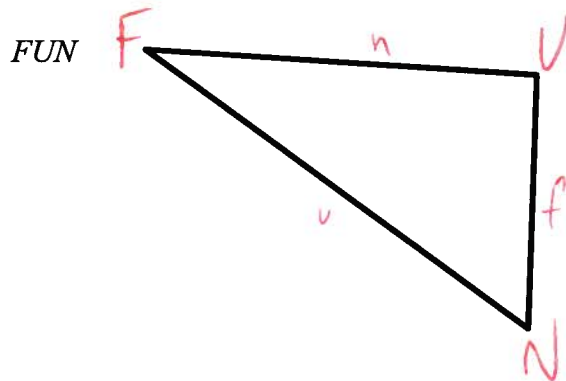


Assignment:

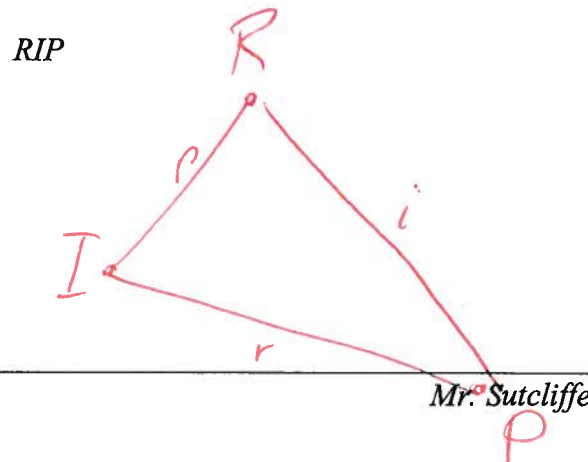
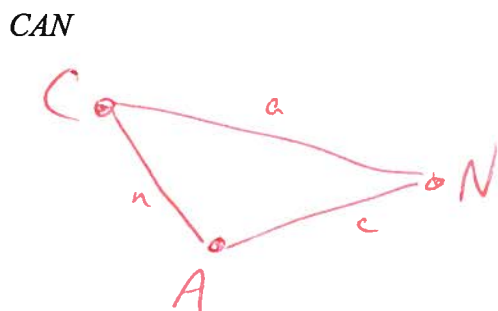
Complete the labels for each triangle. Make sure each pair of opposite sides/angles is matched with the same letter.



Label the following triangles with the given letters:



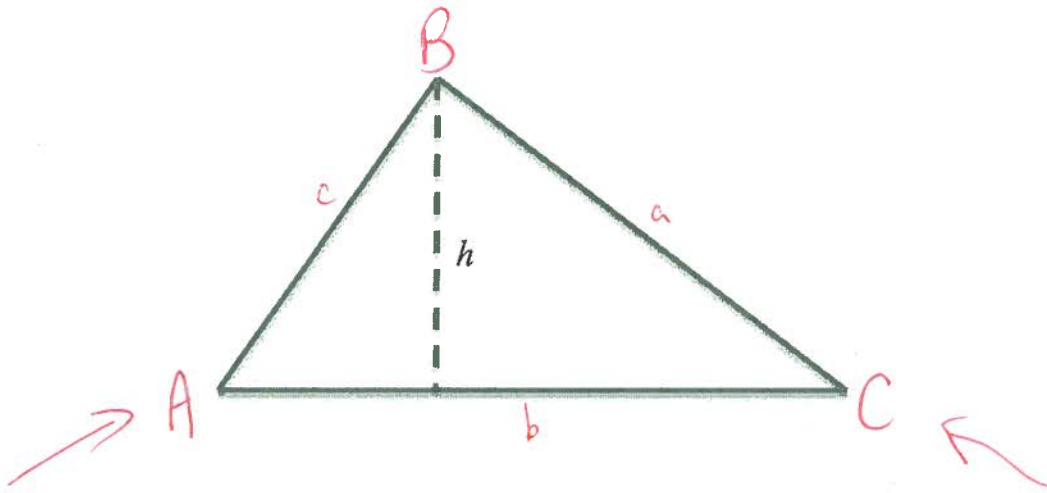
Create your own unique triangles and label them with the following letters



Part 2 – Where does sine law formula come from? Why does it work?

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine law is a short cut to figure out this type of question without using SOH CAH TOA. The sine law is related to the height of a triangle.



$$\sin A = \frac{h}{c}$$

$$\sin C = \frac{h}{a}$$

$$c \sin A = h$$

$$a \sin C = h$$

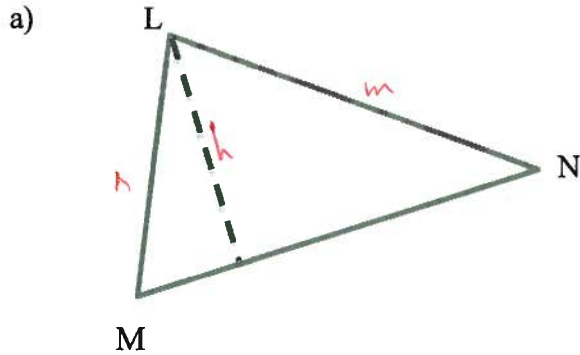
$$c \sin A = a \sin C$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

proved part of the "formula"

Assignment

Follow the steps on the previous page of the notes to create equivalent ratios for these triangles:

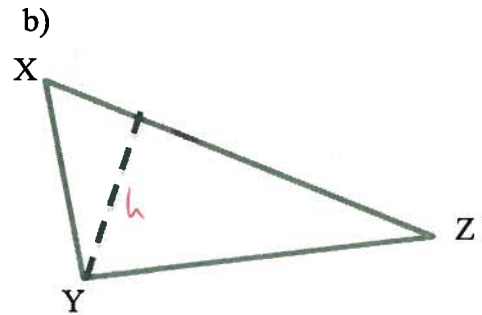


$$\sin M = \frac{h}{n} \quad \sin N = \frac{h}{m}$$

$$n \sin M = h \quad m \sin N = h$$

$$n \sin M = m \sin N$$

$$\frac{n}{\sin N} = \frac{m}{\sin M}$$



$$\sin X = \frac{h}{z} \quad \sin Z = \frac{h}{x}$$

$$z \sin X = h \quad x \sin Z = h$$

$$z \sin X = x \sin Z$$

$$\frac{z}{\sin Z} = \frac{x}{\sin X}$$

Part 3 – How to set up the sine law ratio:

On the test and final exam, the sine law formula will be written like this:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

← To find an unknown side

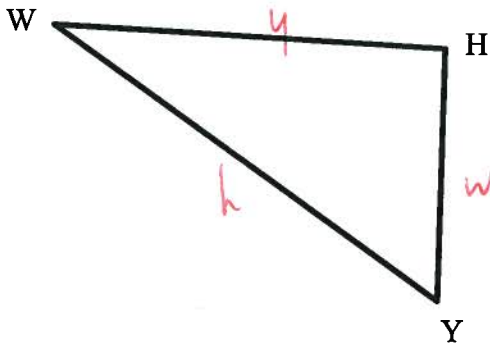
But it can also be written upside-down:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

← To find an unknown angle

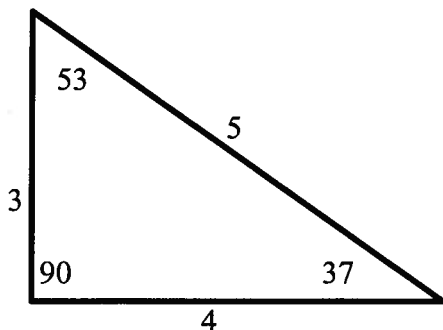
And it can be made specific to the letters and numbers in the question:

Example:



$$\frac{w}{\sin W} = \frac{h}{\sin H} = \frac{y}{\sin Y}$$

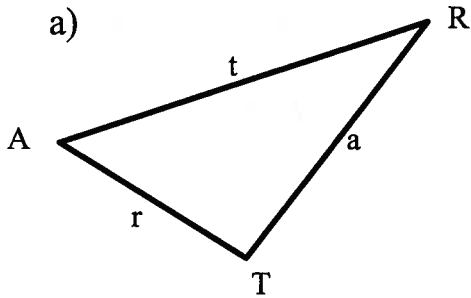
Example:



$$\frac{5}{\sin 90} = \frac{3}{\sin 37} = \frac{4}{\sin 53}$$

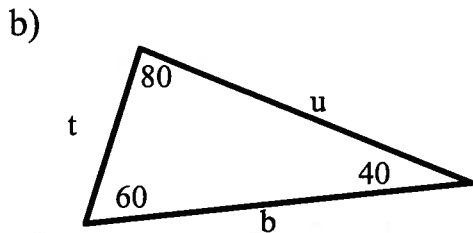
Assignment:

1) Write the sine law ratio for each of the following triangles



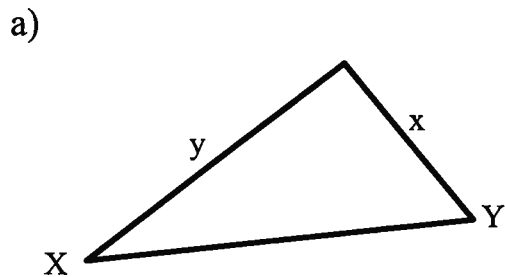
$$\frac{\sin A}{a} = \frac{\sin R}{r} = \frac{\sin T}{t}$$

either way

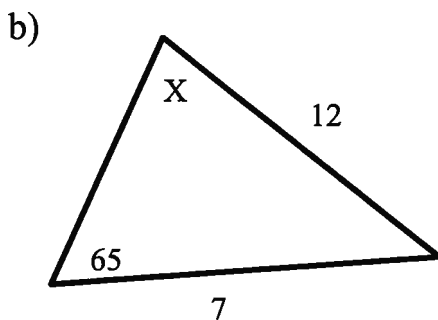


$$\frac{t}{\sin 60} = \frac{u}{\sin 40} = \frac{b}{\sin 80}$$

2) Write the sine law just for the two ratios given in the question:



$$\frac{x}{\sin X} = \frac{y}{\sin Y}$$



$$\frac{12}{\sin 65} = \frac{7}{\sin X}$$

should be on top!

$$\frac{\sin 65}{12} = \frac{\sin X}{7}$$

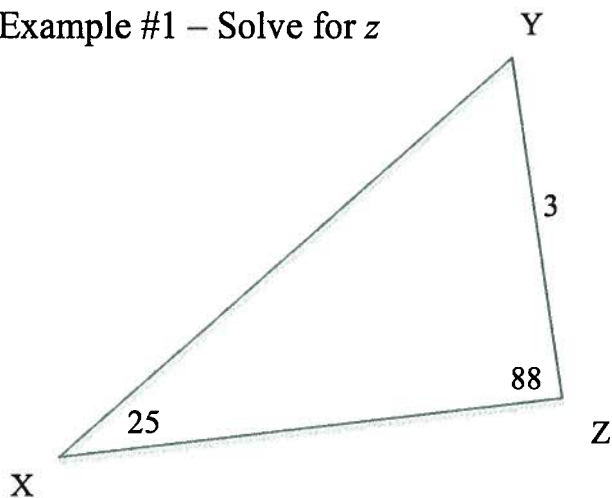
To use the sine law formula, you do not need all three ratios:

Step #1 - Put the unknown value on the top of the first ratio and its opposite side/angle on the bottom

Step #2 - Use the pair of opposites that are given in the question as the other ratio.

Step #3 - Solve for the unknown

Example #1 - Solve for z



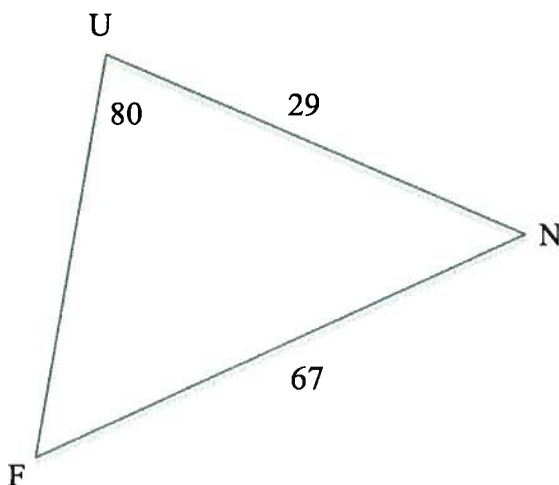
$$\frac{z}{\sin Z} = \frac{x}{\sin X}$$

$$\frac{z}{\sin 88} = \frac{3}{\sin 25}$$

$$z = \frac{3 \sin 88}{\sin 25}$$

$$z = 7.1$$

Example #2 - Solve for F



$$\frac{\sin F}{29} = \frac{\sin 80}{67}$$

$$\sin F = \frac{29 \sin 80}{67}$$

$$\sin F = 0.42626$$

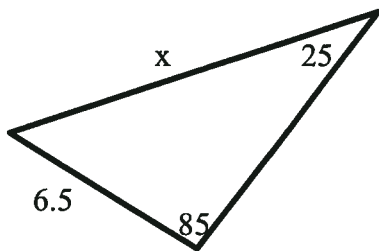
$$F = 25.2^\circ$$

Assignment:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

1) Solve for the unknown side length or angle. Round your answer to one decimal place.

a)

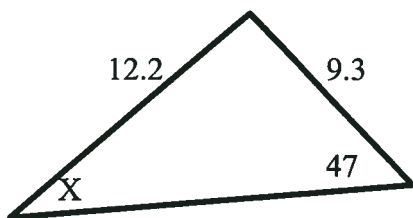


$$\frac{x}{\sin 85} = \frac{6.5}{\sin 25}$$

$$x = \frac{6.5 \sin 85}{\sin 25}$$

$$x = 15.3$$

b)

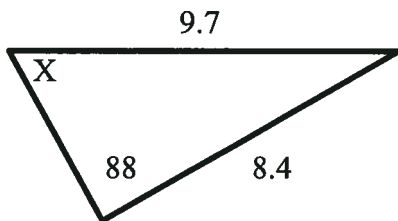


$$\frac{\sin X}{9.3} = \frac{\sin 47}{12.2}$$

$$\sin X = \frac{9.3 \sin 47}{12.2}$$

$$X = 33.9^\circ$$

c)

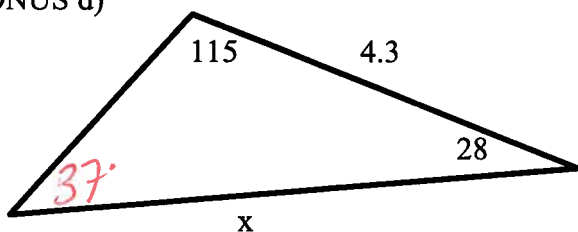


$$\frac{\sin X}{8.4} = \frac{\sin 88}{9.7}$$

$$\sin X = \frac{8.4 \sin 88}{9.7}$$

$$X = 59.9^\circ$$

BONUS d)



$$\frac{x}{\sin 115} = \frac{4.3}{\sin 37}$$

$$x = \frac{4.3 \sin 115}{\sin 37}$$

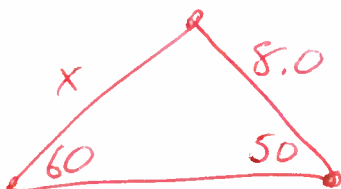
$$x = 6.5$$

Answers

- a) 15.3 b) 33.9 c) 59.9 d) 6.5

2) Sketch a triangle that corresponds to each equation below. Then solve for the unknown side length or angle. Round your answer to one decimal place.

a) $\frac{x}{\sin 50^\circ} = \frac{8.0}{\sin 60^\circ}$



$$x = \frac{8 \sin 50}{\sin 60}$$

$$x = 7.1$$

b) $\frac{\sin X}{6.0} = \frac{\sin 72^\circ}{10.0}$

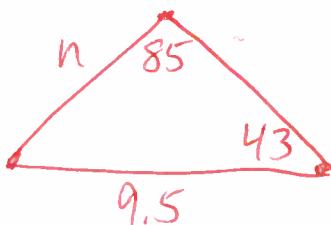


$$\sin X = \frac{6 \sin 72}{10}$$

$$\sin X = 0.5706339$$

$$X = 34.8^\circ$$

c) $\frac{n}{\sin 43^\circ} = \frac{9.5}{\sin 85^\circ}$



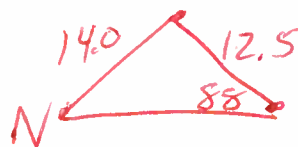
$$n = \frac{9.5 \sin 43}{\sin 85}$$

$$n = 6.5$$

BONUS d) $\frac{12.5}{\sin N} = \frac{14.0}{\sin 88^\circ}$

Flip 1st!

$$\frac{\sin N}{12.5} = \frac{\sin 88}{14.0}$$



$$\sin N = \frac{12.5 \sin 88}{14}$$

$$N = 63.2^\circ$$

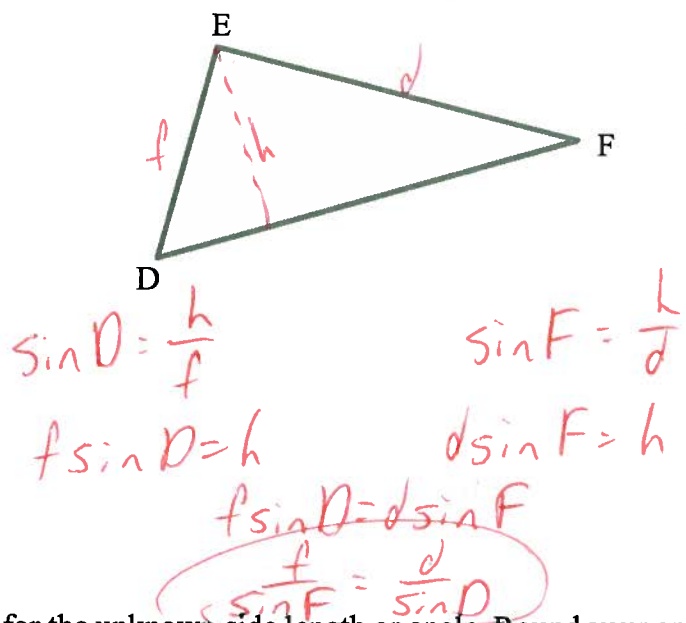
Answers

a) 7.1 b) 34.8 c) 6.5 d) 63.2

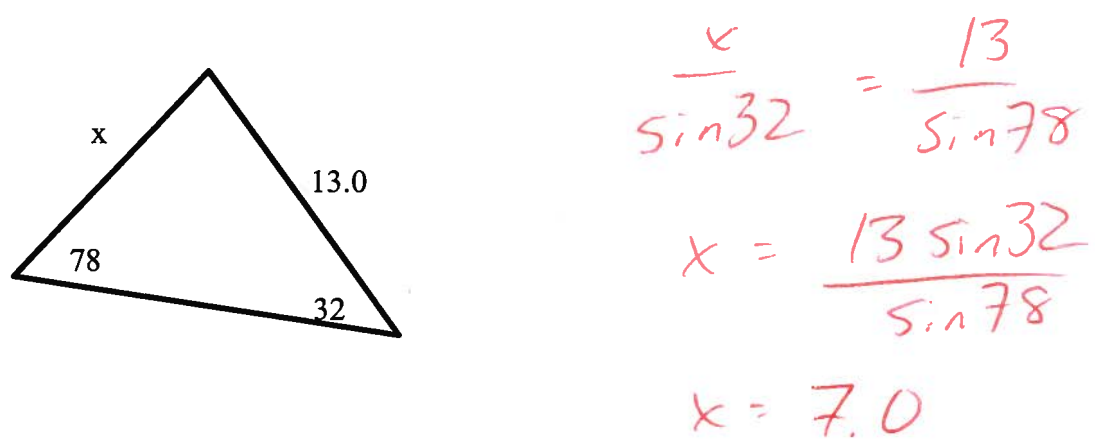
Practice Quiz

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

1) Using basic trig ratios and the height of the triangle, write the steps to prove the Sine Law for the following triangle



2) Solve for the unknown side length or angle. Round your answer to one decimal place.



Answers

- 1) see notes "Part 2" and change ABC to DEF
- 2) 7.0