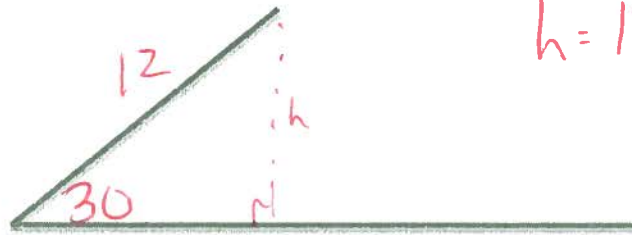


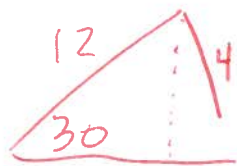
### 3.6 Sine Law: The Ambiguous Case

Say you have a triangle where  $A=30$  and  $b=12$ ... What is the height?



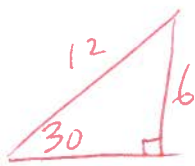
$$h = 12 \sin 30 = 6$$

What happens if  $a$  is 4?



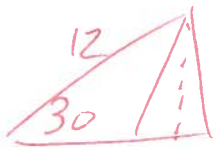
4 is not long enough to make a triangle

What happens if  $a$  is 6?



6 is long enough to make one RIGHT triangle

What happens if  $a$  is 8?



because 8 is longer than the height you can make 2 different triangles.

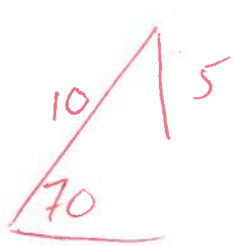
What happens if  $a$  is 15?



because 15 is longer than the adjacent side, you can only make ONE possible triangle

**Example:** Determine if there are zero, one, or two possibilities for this triangle:

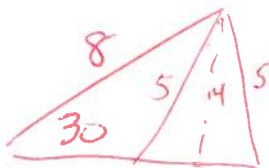
a)  $\angle A = 70^\circ, a = 5 \text{ m}, b = 10 \text{ m}$



$$h = 10 \sin 70 = 9.4$$

No possible triangle

b)  $\angle A = 30^\circ, a = 5 \text{ m}, b = 8 \text{ m}$



$$h = 8 \sin 30 = 4$$

Two possible triangles

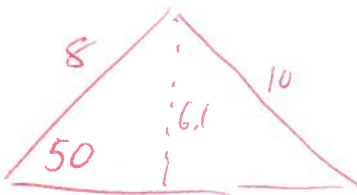
**Example:** Does this triangle involve the SSA situation? How many triangles are possible?

a)  $\angle B = 70^\circ, a = 5 \text{ m}, c = 10 \text{ m}$



SAS - one possible triangle

b)  $\angle C = 50^\circ, a = 8 \text{ m}, c = 10 \text{ m}$  ASS - Ambiguous Case

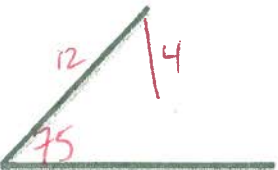

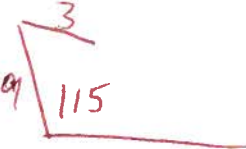
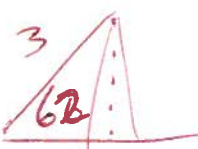


$$h = 8 \sin 50 = 6.1$$


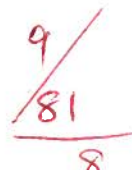
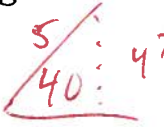
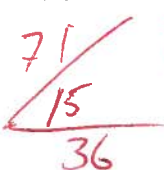
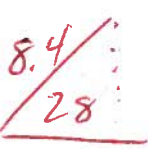
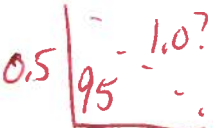
one possible triangle

**Assignment:**

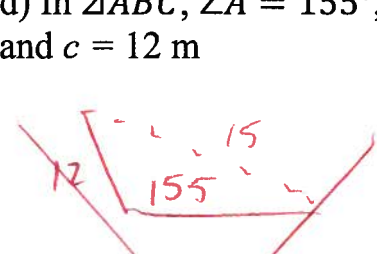
Given each set of measurements for  $\triangle ABC$ , determine if there are zero, one, or two possibilities. Draw the triangle(s) to support your answer.

<p>1a) <math>\angle A = 75^\circ</math>, <math>a = 4</math> m, and <math>b = 12</math> m</p>  <p><math>h = 12 \sin 75 = 11.6</math></p> <p>No solution (4 is not longer than height of 11.6)</p>	<p>1b) <math>\angle A = 50^\circ</math>, <math>a = 10</math> m, and <math>b = 6</math> m</p>  <p><math>h = 6 \sin 50 = 4.6</math></p> <p>one solution (10 is longer than adjacent side 6)</p>
<p>1c) <math>\angle A = 115^\circ</math>, <math>a = 3.0</math> m, and <math>b = 9.0</math> m</p>  <p>No solution (3 won't be long enough)</p>	<p>1d) <math>\angle A = 62^\circ</math>, <math>a = 2.8</math> m, and <math>b = 3.0</math> m</p>  <p><math>h = 3 \sin 62 = 2.6</math></p> <p>two solutions (because 2.8 is between 2.6 and 3.0)</p>

- 2) Decide whether each description of a triangle involves the SSA situation.  
 3) Then calculate the height of each triangle and determine the number of triangles that are possible (zero, one, or two). Justify your answers.

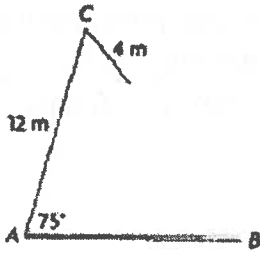
<p>a) In <math>\triangle ABC</math>, <math>\angle B = 100^\circ</math>, <math>a = 8</math> cm, and <math>b = 10</math> cm</p>  <p>height won't help!</p> $\frac{\sin X}{8} = \frac{\sin 100}{10}$ <p><math>X = 52^\circ</math> One Answer              (can't have 2 angles <math>&gt; 90</math> in triangle)</p>	<p>b) In <math>\triangle DEF</math>, <math>\angle D = 81^\circ</math>, <math>e = 9</math> cm, and <math>f = 8</math> cm</p>  <p>SAS NOT SSA!</p> <p>one answer</p>
<p>c) In <math>\triangle GHI</math>, <math>\angle G = 40^\circ</math>, <math>i = 5</math> cm, and <math>g = 4</math> cm</p>  <p><math>h = 5 \sin 40</math>  <math>h = 3.2</math></p> <p>Two Answers              (4 is between 3.2 + 5.0)</p>	<p>d) In <math>\triangle JKL</math>, <math>\angle L = 15^\circ</math>, <math>j = 71</math> cm, and <math>k = 36</math> cm</p>  <p>SAS NOT SSA</p> <p>one answer</p>
<p>e) In <math>\triangle MNO</math>, <math>\angle O = 28^\circ</math>, <math>m = 8.4</math> cm, and <math>o = 4.0</math> cm</p>  <p><math>h = 8.4 \sin 28</math>  <math>h = 3.9</math></p> <p>Two Answers (barely)              Close to Right Triangle</p>	<p>f) In <math>\triangle PQR</math>, <math>\angle Q = 95^\circ</math>, <math>q = 1.0</math> cm, and <math>r = 0.5</math> cm</p>  $\frac{\sin X}{0.5} = \frac{\sin 95}{1.0}$ <p><math>X = 29.9</math> One Answer</p>

4) Decide whether each description of a triangle involves the SSA situation. If it does, determine the number of triangles (zero, one, or two) that are possible with the given measurement. Draw the triangle(s) and justify your answer.

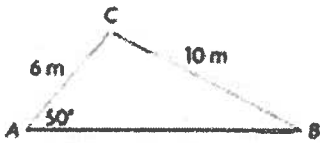
<p>a) In <math>\triangle ABC</math>, <math>\angle A = 51^\circ</math>, <math>a = 5</math> m, and <math>b = 14</math> m</p> <p><math>14 / 51</math> 5? <math>h = 14 \sin 51 = 10.9</math></p> <p><u>51</u></p> <p>No Triangle (10.9 is bigger than 5)</p>	<p>b) In <math>\triangle ABC</math>, <math>\angle C = 30^\circ</math>, <math>a = 6</math> mm, and <math>c = 12</math> mm</p> <p><math>6 / 30</math> 12?</p> <p><u>30</u></p> <p>One Triangle (12 is larger than 6)</p>
<p>c) In <math>\triangle ABC</math>, <math>\angle B = 40^\circ</math>, <math>a = 12</math> cm, and <math>b = 10</math> cm</p> <p><math>12 / 40</math> 10? <math>12 \sin 40 = 7.7</math></p> <p><u>40</u></p> <p>Two Triangles (because 10 is between 7.7 and 12)</p>	<p>d) In <math>\triangle ABC</math>, <math>\angle A = 155^\circ</math>, <math>b = 15</math> m, and <math>c = 12</math> m</p> <p></p> <p><math>\frac{\sin X}{12} = \frac{\sin 155}{15}</math></p> <p><math>X = 19.8</math></p> <p>One Answer</p> <p>Not SSA</p>

Answer Key

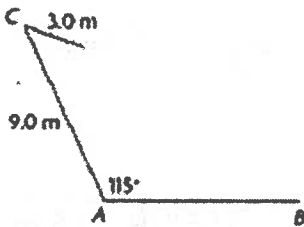
1. a) zero



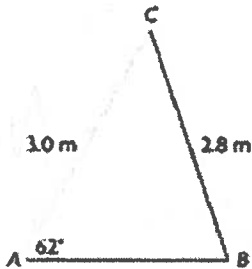
b) one



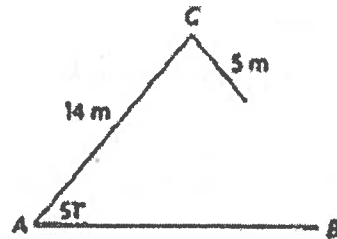
c) zero



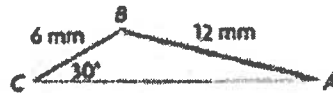
d) two



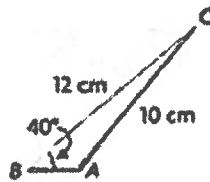
2. a) SSA  
 b) not SSA  
 c) SSA  
 d) not SSA  
 e) SSA  
 f) SSA
3. a)  $h = 7.9$  cm, one  
 b)  $h = 7.9$  cm or  $h = 8.9$  cm, one  
 c)  $h = 3.2$  cm, two  
 d)  $h = 9.3$  cm or  $h = 18.4$  cm, one  
 e)  $h = 3.9$  cm, two  
 f)  $h = 0.5$  cm, one
4. a) SSA, zero



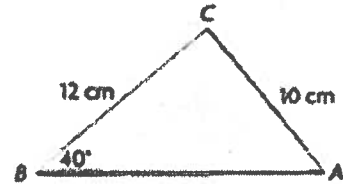
b) SSA, one



c) SSA, two



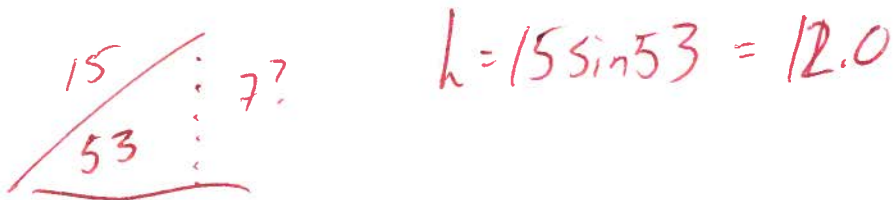
d) not SSA



## Practice Quiz

Decide whether each description of a triangle involves the SSA situation. If it does, determine the number of triangles (zero, one, or two) that are possible with the given measurement. Draw the triangle(s) and justify your answer.

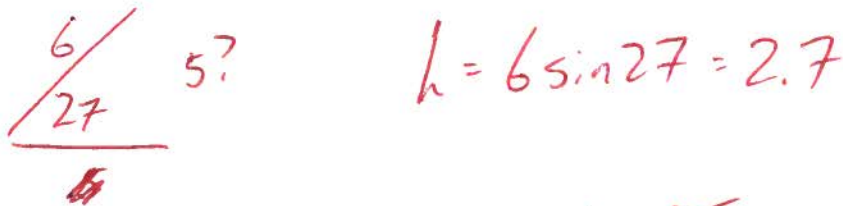
- 1) In  $\triangle ABC$ ,  $\angle A = 53^\circ$ ,  $a = 7$  m, and  $b = 15$  m



No Triangle Possible

$$\frac{\sin X}{15} = \frac{\sin 53}{7} \quad X = \text{error}$$

- 2) In  $\triangle ABC$ ,  $\angle A = 27^\circ$ ,  $a = 5$  m, and  $b = 6$  cm



Two Possible Triangles  
(because 5 is between 2.7 and 6)

Answers: 1) 0 2) 2

