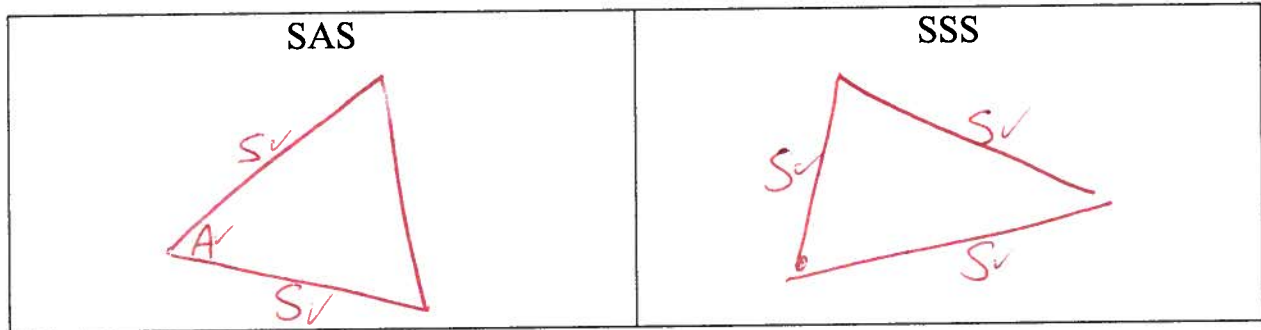


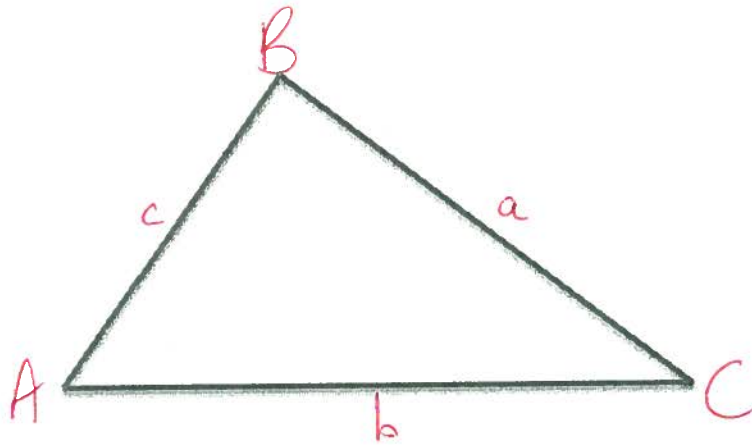
Sutcliffe

3.7 Cosine Law

For non-right triangles that can't be solved with the sine law



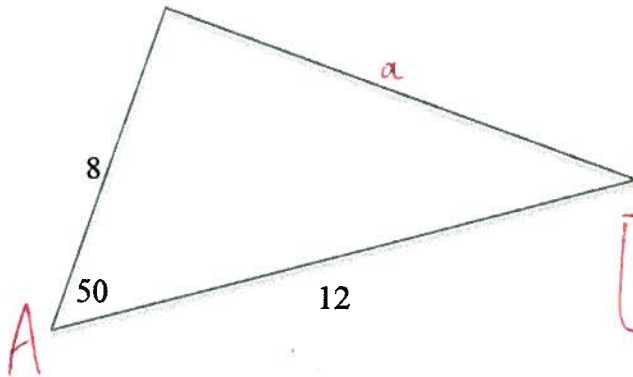
Cosine Law can be proven based on the Pythagorean Theorem



$$a^2 = b^2 + c^2 - 2bc \cos A$$

adjustment because
it's not a RIGHT triangle.

Example #1:



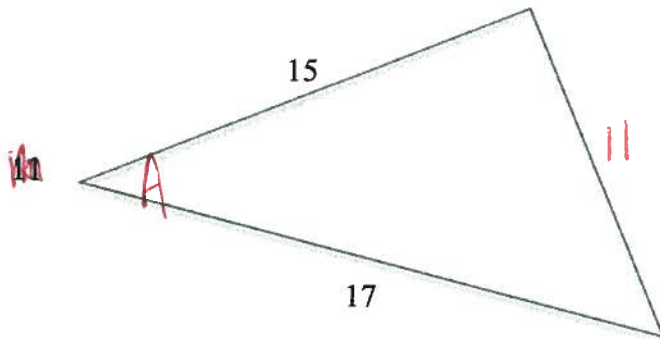
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 8^2 + 12^2 - 2(8)(12)\cos 50$$

$$a^2 = 84.6$$

$$a = 9.2$$

Example #2:



$$11^2 = 15^2 + 17^2 - 2(15)(17)\cos A$$

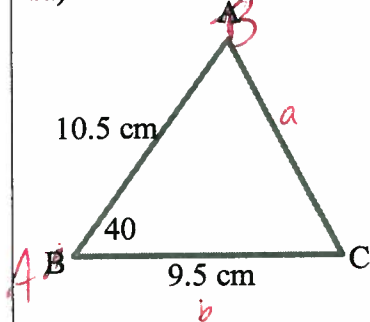
$$\frac{11^2 - 15^2 - 17^2}{-2(15)(17)} = \cos A$$

$$0.7706 = \cos A$$

$$39.6^\circ = A$$

Assignment:

1a) Find the unknown side length



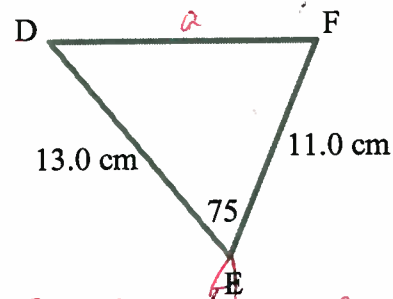
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 9.5^2 + 10.5^2 - 2(9.5)(10.5) \cos 40$$

$$a^2 = 47.7$$

$$a = 6.9 \text{ cm}$$

1b) Find the unknown side length



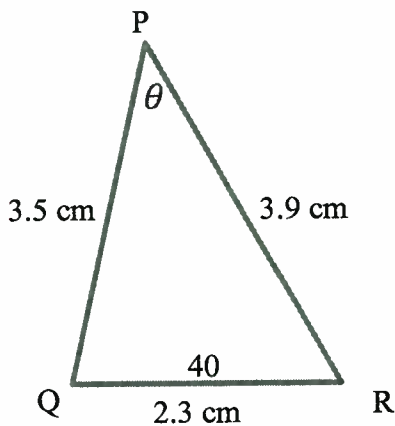
$$e^2 = d^2 + f^2 - 2df \cos E$$

$$e^2 = 13^2 + 11^2 - 2(13)(11) \cos 75$$

$$e^2 = 215.98$$

$$e = 14.7$$

2a) Find the indicated angle



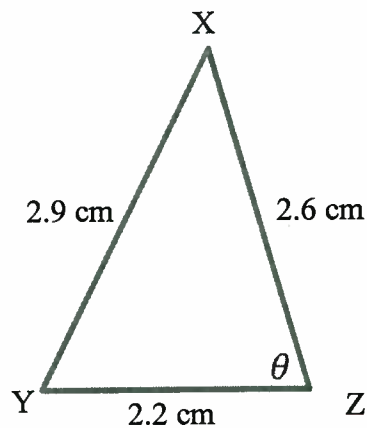
$$2.3^2 = 3.5^2 + 3.9^2 - 2(3.5)(3.9) \cos \theta$$

$$\frac{2.3^2 - 3.5^2 - 3.9^2}{-2(3.5)(3.9)} = \cos \theta$$

$$.8121 = \cos \theta$$

$$\theta = 35.7^\circ$$

2b) Find the indicated angle



$$2.9^2 = 2.2^2 + 2.6^2 - 2(2.2)(2.6) \cos \theta$$

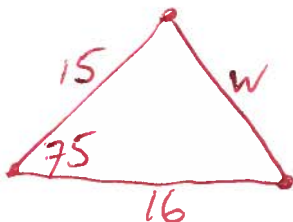
$$\frac{2.9^2 - 2.2^2 - 2.6^2}{-2(2.2)(2.6)} = \cos \theta$$

$$.2788 = \cos \theta$$

$$73.8^\circ = \theta$$

3a) Sketch the triangle and find the unknown

$$w^2 = 15^2 + 16^2 - 2(15)(16)\cos 75^\circ$$

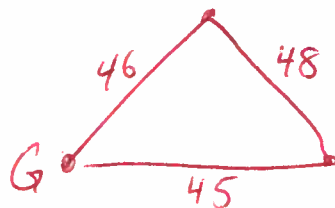


$$w^2 = 356.8$$

$$w = 18.9$$

3b) Sketch the triangle and find the unknown

$$48^2 = 46^2 + 45^2 - 2(46)(45)\cos G$$



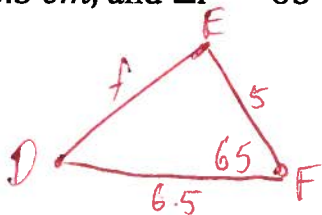
$$\frac{48^2 - 46^2 - 45^2}{-2(46)(45)} = \cos G$$

$$.4437 = \cos G$$

$$G = 63.7^\circ$$

4a) Solve the triangle:

In $\triangle DEF$, $d = 5.0$ cm, $e = 6.5$ cm, and $\angle F = 65^\circ$



$$f^2 = 6.5^2 + 5^2 - 2(6.5)(5)\cos 65$$

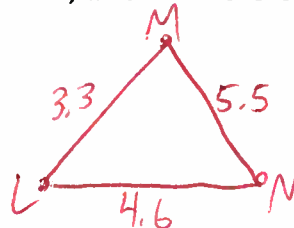
$$f = 6.3$$

$$\frac{\sin D}{5} = \frac{\sin 65}{6.5} \quad D = 45.9^\circ$$

$$E = 180 - 65 - 45.9 = 69.1^\circ$$

4b) Solve the triangle:

In $\triangle LMN$, $l = 5.5$ cm, $m = 4.6$ cm, and $n = 3.3$ cm



$$5.5^2 = 3.3^2 + 4.6^2 - 2(3.3)(4.6)\cos L$$

$$\frac{5.5^2 - 3.3^2 - 4.6^2}{-2(3.3)(4.6)} = \cos L \quad L = 86.6$$

$$\frac{\sin N}{3.3} = \frac{\sin 86.6}{5.5} \quad N = 36.8$$

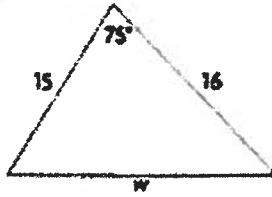
$$M = 180 - 86.6 - 36.8 = 56.6^\circ$$

Answer Key

1) a) 6.9 cm b) 14.7 cm

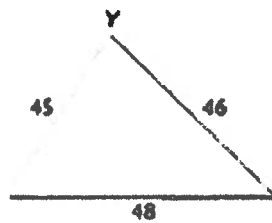
2) a) 35.7 b) 74

3) a)



$$w = 18.9$$

b)



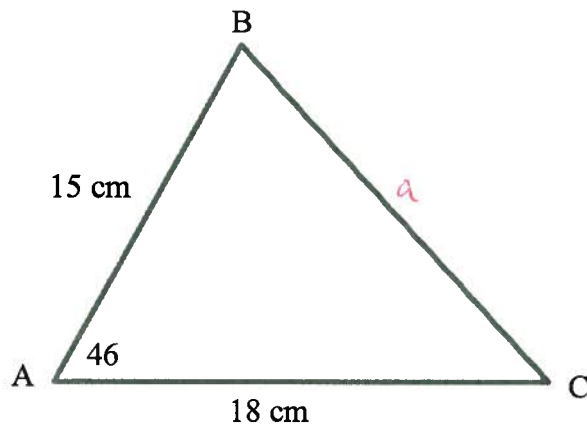
$$\angle Y = 63.7^\circ$$

4)

a) $f = 6.3 \text{ cm}$, $\angle D = 45.9^\circ$, $\angle E = 69.1^\circ$ b) $\angle L = 86.6^\circ$, $\angle M = 56.6^\circ$, $\angle N = 36.8^\circ$

Practice Quiz

1) Use the Cosine Law to find the measure of side a .

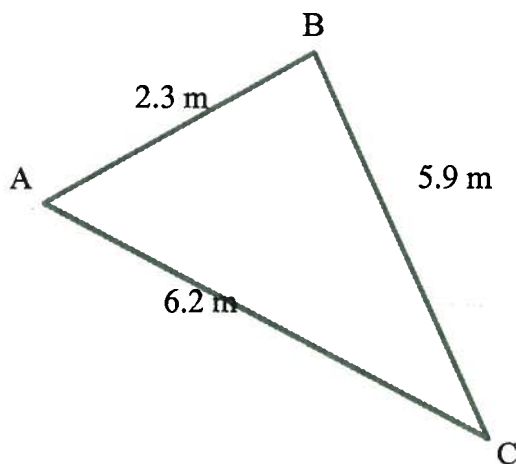


$$a^2 = 15^2 + 18^2 - 2(15)(18)\cos 46$$

$$a^2 = 173.88$$

$$a = 13.2 \text{ cm}$$

2) Use the Cosine Law to find the measure of angle A.



$$5.9^2 = 2.3^2 + 6.2^2 - 2(2.3)(6.2)\cos A$$

$$\frac{5.9^2 - 2.3^2 - 6.2^2}{-2(2.3)(6.2)} = \cos A$$

$$0.3128 = \cos A$$

$$A = 71.8^\circ$$

Answers: 1) 13 cm 2) 72°