

### Chapter 3: Trigonometry

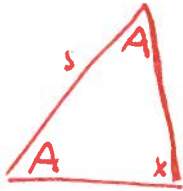
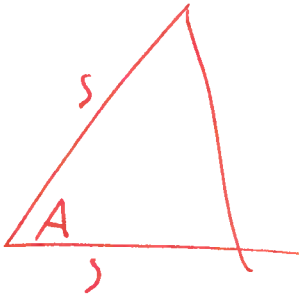
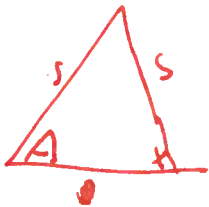
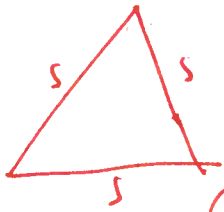
#### 3.10 Sine or Cosine?

Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Law

$$c^2 = a^2 + b^2 - 2ab \cos C$$

<p>ASA or AAS</p>  <p>Sine Law</p>	<p>SAS</p>  <p>Cosine Law</p>
<p>ASS</p> <p>Ambiguous?</p>  <p>Sine Law</p>	<p>SSS</p>  <p>Cosine Law</p>

EXERCISES

PROBLEM 1

QUESTION

ANSWER

1. The area of a triangle is  $\frac{1}{2}bh$ . If the base is 10 units and the height is 5 units, what is the area?

2. A right-angled triangle has a hypotenuse of 13 units and one leg of 5 units. What is the length of the other leg?

3. The area of a parallelogram is 48 square units. If the base is 8 units, what is the height?

4. A trapezoid has a top base of 4 units, a bottom base of 10 units, and a height of 6 units. What is its area?

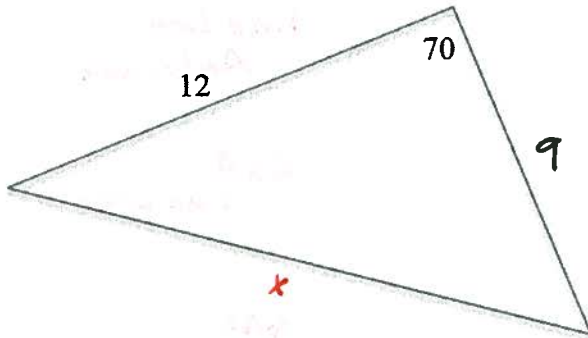


PROBLEM 2

QUESTION



Example #1:



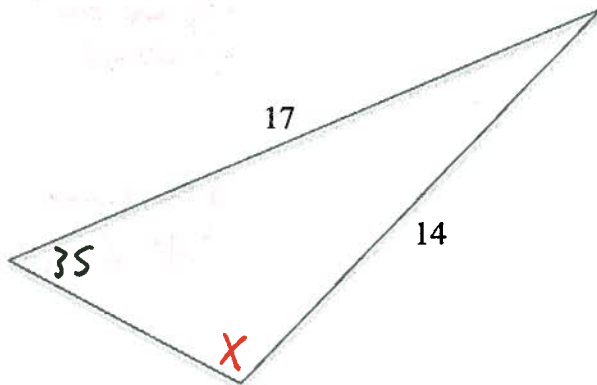
SAS - Cosine Law

$$x^2 = 12^2 + 9^2 - 2(12)(9)\cos 70$$

$$x^2 = 151.$$

$$x = 12.3$$

Example #2:

ASS - Sine Law  
Ambiguous?

$$\frac{\sin x}{17} = \frac{\sin 35}{14}$$

$$x = 44^\circ$$

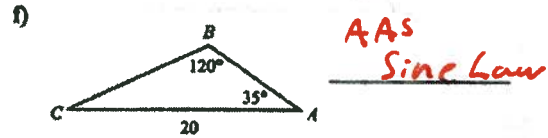
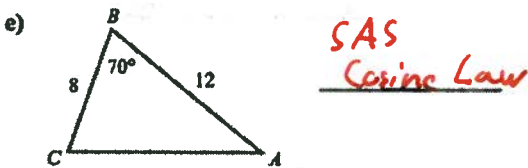
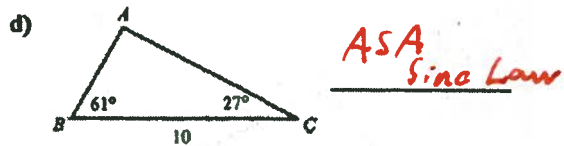
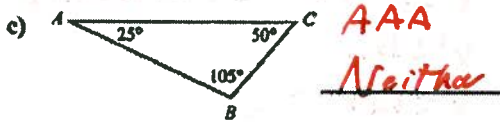
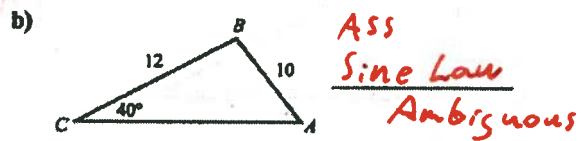
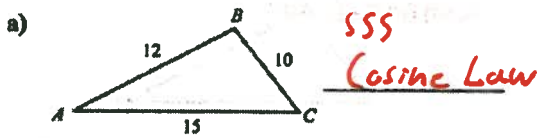
Other Answer

~~44~~

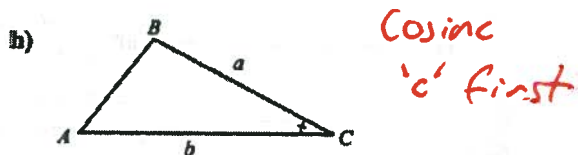
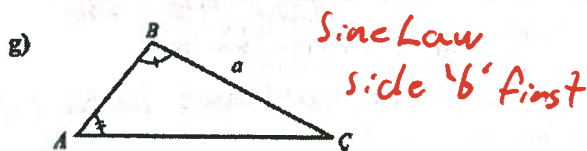
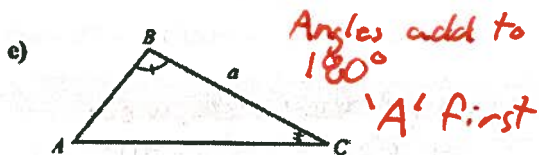
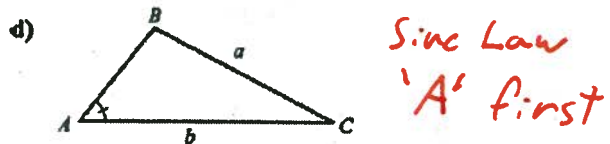
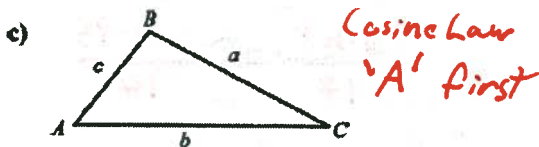
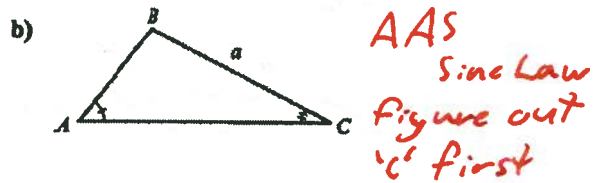
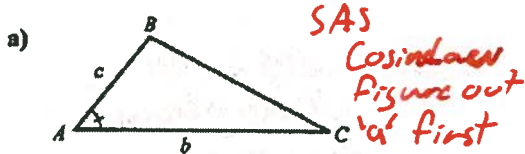
$$180 - 44 = 136^\circ$$

picture looks like 136°

1) Determine whether the Law of Sines or the Law of Cosines would be used to begin the solution process for each triangle.

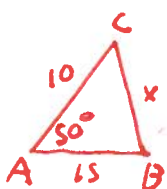


2) Given the indicated parts of  $\triangle ABC$ , what angle or side should be found first, and which formula should be used to find it?



3) Solve each triangle ABC. Round answers to one decimal place:

a)  $\angle A = 50^\circ, b = 10, c = 15$   $\angle B = 47.8^\circ, \angle C = 82.2^\circ, a = 11.5$



SAS - Cosine Law

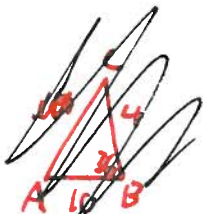
$$x^2 = 10^2 + 15^2 - 2(10)(15) \cos 50$$

$$x = 11.5$$

$$\frac{\sin B}{10} = \frac{\sin 50}{11.5}$$

$$180 - 47.8 - 50 = 82.2 \quad B = 47.8$$

b)  $\angle B = 36^\circ, a = 4, c = 10$   ~~$\angle A = 17.6^\circ, \angle C = 130.4^\circ$~~  SAS - Cosine



~~ASS - Sine Law Ambiguous?~~

~~$$\frac{\sin A}{4} = \frac{\sin 36}{10} \cdot 4$$~~

~~$$A = 17.6^\circ$$~~

~~$$180 - 17.6 - 36 = 126.4^\circ$$~~

$$\angle A = 19.1^\circ, \angle C = 124.9^\circ$$

$$b = 7.2$$

c)  $\angle C = 60^\circ, b = 4, a = 8$

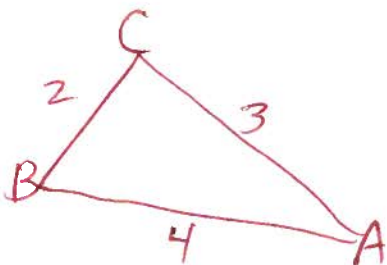


SAS - Cosine Law

$$c^2 = 4^2 + 8^2 - 2(4)(8) \cos 60 \quad c = 6.9$$

$$\frac{\sin A}{8} = \frac{\sin 60}{6.9} \quad A = 90^\circ \quad B = 30^\circ$$

d)  $a = 2, b = 3, c = 4$



SSS - Cosine Law

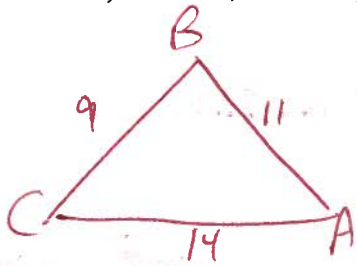
$$2^2 = 3^2 + 4^2 - 2(3)(4) \cos A$$

$$A = 28.96 = 29.0$$

$$\frac{\sin B}{3} = \frac{\sin 29}{2} \quad B = 46.7^\circ$$

$$C = 104.3^\circ$$

e)  $a = 9, b = 14, c = 11$



SSS-Cosine Law

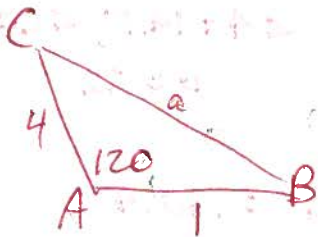
$$9^2 = 11^2 + 14^2 - 2(11)(14)\cos A$$

$$A = 40^\circ$$

$$\frac{\sin C}{11} = \frac{\sin 40}{9} \quad C = 51.8^\circ$$

$$B = 88.2^\circ$$

f)  $b = 4, c = 1, \angle A = 120^\circ$



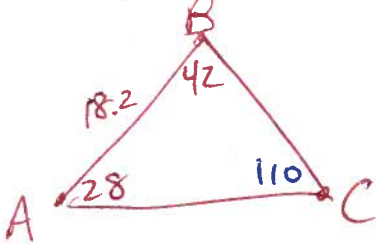
SAS-Cosine Law

$$a^2 = 4^2 + 1^2 - 2(4)(1)(\cos 120)$$

$$a = 4.6$$

$$\frac{\sin B}{4} = \frac{\sin 120}{4.6} \quad B = 48.9^\circ \quad C = 11.1^\circ$$

g)  $\angle A = 28^\circ, \angle B = 42^\circ, c = 18.2$



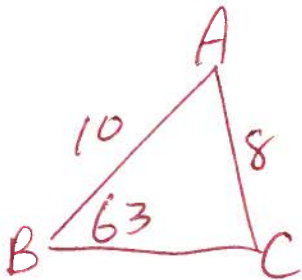
ASA-Sine Law

$$C = 180 - 42 - 28 = 110$$

$$\frac{b}{\sin 42} = \frac{18.2}{\sin 110} = 13.0$$

$$\frac{a}{\sin 28} = \frac{18.2}{\sin 110} \quad a = 9.1$$

h)  $\angle B = 63^\circ, b = 8, c = 10$

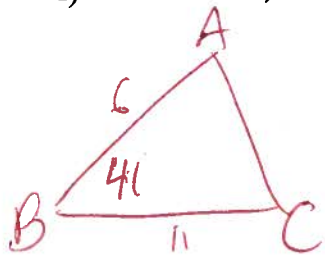


ASS-Sine Law

$$\frac{\sin C}{10} = \frac{\sin 63}{8} \quad \text{error}$$

No solution

i)  $\angle B = 41^\circ, a = 11, c = 6$



SAS = Cosine Law

$$b^2 = 6^2 + 11^2 - 2(6)(11)\cos 41$$

$$b = 7.6$$

$$\frac{\sin A}{11} = \frac{\sin 41}{7.6}$$

$$A = 76.7 \text{ OR } 108.3^\circ$$

$$\frac{\sin C}{6} = \frac{\sin 41}{7.6}$$

~~$$C = 67.3^\circ$$~~

$$C = 31.3^\circ$$

### Answer Key

1a) Cosine Law b) Sine Law c) Neither d) Sine Law e) Cosine Law f) Sine Law

2a) Cosine Law: side  $a$  b) Sine Law: side  $c$  c) Cosine Law: angle  $B$  d) Sine Law: angle  $B$   
 e) Angles add to 180: angle  $A$  f) Can't be solved g) Sine Law: side  $b$  h) Cosine Law: side  $c$

3) a)  $\angle B = 41.8^\circ, \angle C = 88.2^\circ, a = 11.5$  b)  $\angle B = 19.2^\circ, \angle C = 124.8^\circ, b = 7.2$   
 c)  $\angle A = 90^\circ, \angle B = 30^\circ, a = 6.9$  d)  $\angle A = 29.0^\circ, \angle B = 46.6^\circ, \angle C = 104.4^\circ$   
 e)  $\angle A = 40.0^\circ, \angle B = 88.2^\circ, \angle C = 51.8^\circ$  f)  $\angle B = 49.1^\circ, \angle B = 10.9^\circ, a = 4.6$   
 g)  $\angle C = 110^\circ, a = 9.1, b = 13.0$  h) No Solution i)  $\angle A = 107.7^\circ, \angle C = 31.3^\circ, b = 7.6$

**Sine or Cosine Word Problems**

## Basic Trig Ratios

SOH CAH TOA

## Geometry Rules

Angles in a triangle = 180

## Sine Law

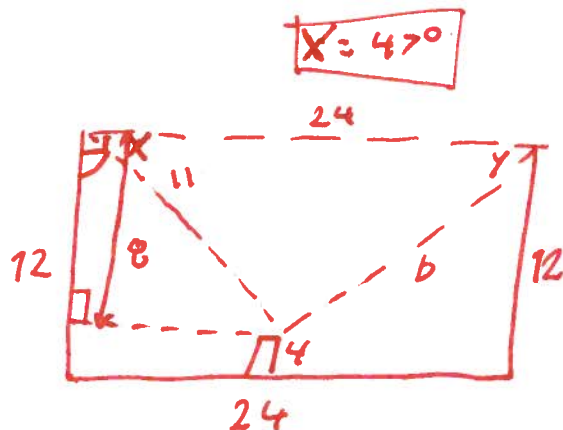
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

## Cosine Law

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**Example #1**

Two security cameras in an expensive jewelry store must be adjusted to monitor an expensive diamond. The cameras are mounted 12 ft. above the floor, directly across from each other on opposite walls. The walls are 24 ft. apart. The diamond is displayed in a case 4 ft high. The distance from the camera on the left to the diamond is 11 ft. Both cameras must aim directly at the diamond. What is the angle of depression for both cameras?



SOH CAH TOA

$$\cos A = \frac{8}{11} \quad A = 43^\circ$$

cosine law

$$b^2 = 11^2 + 24^2 - 2(11)(24) \cos 47^\circ$$

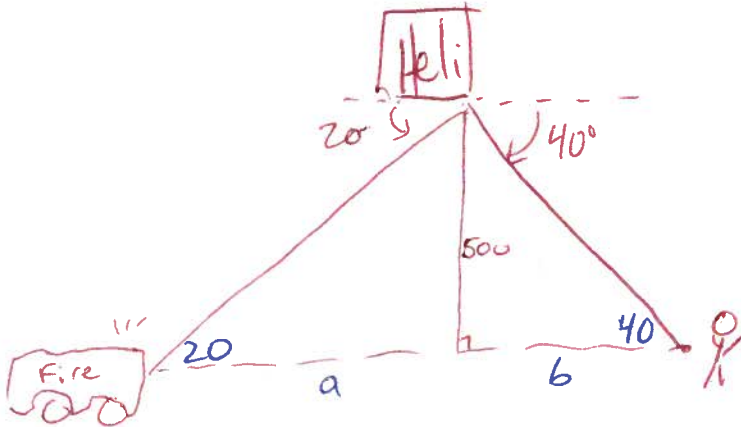
$$b = 18.4$$

$$\frac{\sin y}{b} = \frac{\sin 47^\circ}{11} \quad \therefore \quad \boxed{y = 26^\circ}$$



**Example #2**

Mr. Seywerd is in a Search and Rescue helicopter flying 500m over Cultus Lake. When he looks directly SE, he sees a person in distress at the bottom of a cliff. The angle of depression to the person is 40 degrees. When he looks back directly NW, he sees a firetruck approaching. The angle of depression to the firetruck is 20 degrees. How far away is the firetruck from the person in distress? If the firetruck approaches with a directional average speed of 70 km/h, how long will it take for the firetruck to reach the scene of the accident?



$$\tan 20 = \frac{500}{a} \quad \tan 40 = \frac{500}{b}$$

$$a = \frac{500}{\tan 20} \quad b = \frac{500}{\tan 40}$$

$$a = 1374 \quad b = 596$$

$$a + b = 1970 \text{ m}$$

$$\frac{70 \text{ km}}{\text{h}} = \frac{1.97 \text{ km}}{?}$$

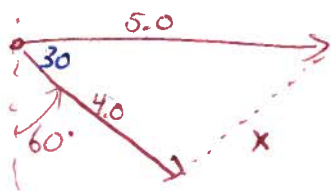
$$0.028 \text{ hours}$$

$$1.69 \text{ minutes}$$

$$1 \text{ minute, } 41 \text{ seconds.}$$

**Assignment:**

1) A kayak leaves Rankin Inlet, Nunavut, and heads due east for 5.0 km, as shown in the diagram. At the same time, a second kayak travels in a direction S60E from the inlet for 4.0km. How far apart, to the nearest tenth of a kilometer, are the kayaks?



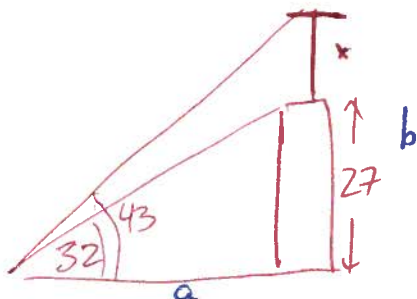
SAS - Cosine Law

$$x^2 = 4^2 + 5^2 - 2(4)(5)\cos 30$$

$$x^2 = 6.36$$

$$x = 2.5 \text{ km}$$

2) A crane stands on top of a building, as shown. How far is the point on the ground from the base of the building, to the nearest tenth of a metre? How tall is the crane?



$$\tan 32 = \frac{27}{a}$$

$$\tan 43 = \frac{b}{43.2}$$

$$a = \frac{27}{\tan 32}$$

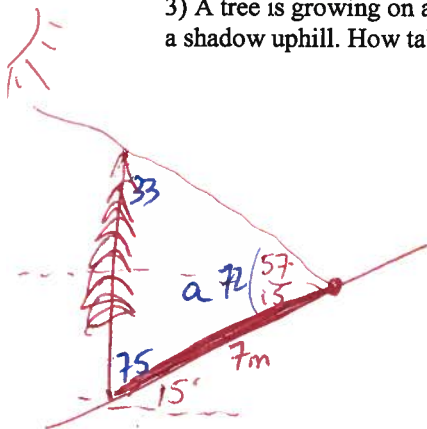
$$b = 43.2 \tan 43$$

$$a = 43.2 \text{ m}$$

$$b = 40.3$$

$$x = 40.3 - 27 = 13.3 \text{ m}$$

3) A tree is growing on a hillside, as shown. The hillside is inclined at an angle of 15 to the horizontal. The tree casts a shadow uphill. How tall is the tree, to the nearest metre?

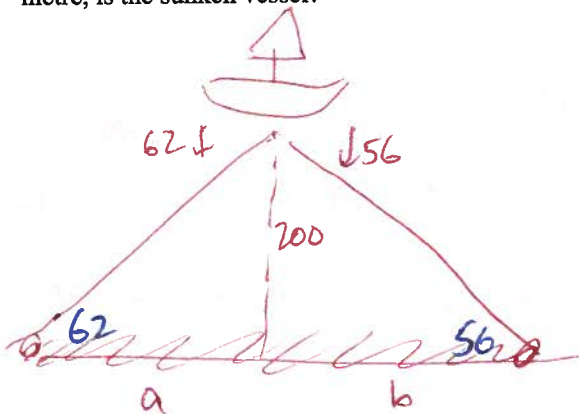


ASA - Sine Law

$$\frac{x}{\sin 72} = \frac{7}{\sin 33}$$

$$x = 12.2 \text{ m}$$

4) A radar operator on a ship discovers a large sunken vessel lying parallel to the ocean surface, 200m directly below the ship. The length of the vessel is a clue to which wreck has been found. The radar operator measures the angles of depression to the front and back of the sunken vessel to be 56 and 62. How long, to the nearest tenth of a metre, is the sunken vessel?



$$\tan 62 = \frac{200}{a}$$

$$\tan 56 = \frac{200}{b}$$

$$a = \frac{200}{\tan 62}$$

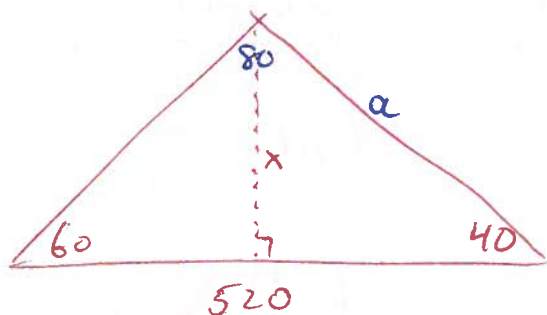
$$b = \frac{200}{\tan 56}$$

$$a = 106.3$$

$$b = 134.9$$

$$a + b = 241.2 \text{ m}$$

5) Fred and Agnes are 520m apart. As Brendan flies overhead in an airplane, they estimate the angle of elevation of the airplane. Fred, looking South, estimates the angle of elevation to be 60. Agnes, looking north, estimates it to be 40. What is the altitude of the airplane, to the nearest tenth of a metre?



$$\frac{a}{\sin 60} = \frac{520}{\sin 80}$$

$$a = 457.3$$

$$\sin 40 = \frac{x}{457.3}$$

$$x = 457.3 \sin 40$$

$$x = 293.9 \text{ m}$$

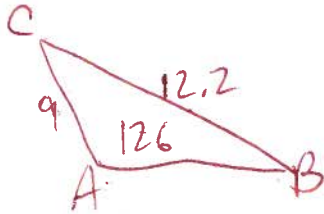
#### Answers

1) 2.5 km 2) 43.2 m, 13.3 m 3) 8 m 4) 241.2 m 5) 293.9 m 5) 879.3m, 40 seconds

**Practice Quiz**

Solve each triangle ABC. Round answers to one decimal place:

1)  $\angle A = 126^\circ, b = 9, a = 12.2$



ASS-Sine Law

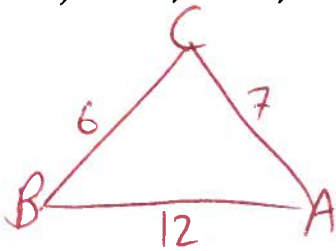
$$\frac{\sin B}{9} = \frac{\sin 126}{12.2} \quad B = 36.6^\circ$$

$$180 - 126 - 36.6$$

$$C = 17.4^\circ$$

$$c^2 = 9^2 + 12.2^2 - 2(9)(12.2)\cos 17.4 \quad c = 4.5$$

2)  $a = 6, b = 7, c = 12$



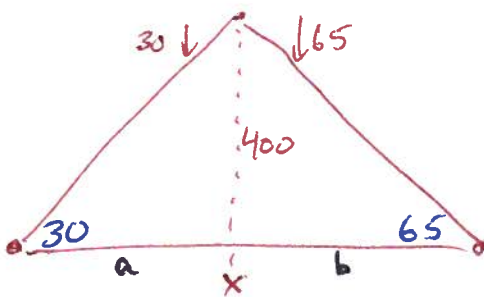
SSS-Cosine Law

$$6^2 = 7^2 + 12^2 - 2(7)(12)\cos A \quad A = 20.8^\circ$$

$$12^2 = 6^2 + 7^2 - 2(6)(7)\cos C \quad C = 134.6^\circ$$

$$180 - 20.8 - 134.6 = B = 24.6^\circ$$

3) Ryan is in a police helicopter, 400m directly above the Sea to Sky highway near Whistler. When he looks north, the angle of depression to a car accident is 65. When he looks south, the angle of depression to the approaching ambulance is 30. How far away from the accident is the ambulance? How long will it take to get to the accident?



$$\tan 30 = \frac{400}{a}$$

$$\tan 65 = \frac{400}{b}$$

$$a = \frac{400}{\tan 30}$$

$$b = \frac{400}{\tan 65}$$

$$a = 692.8$$

$$b = 186.5$$

$$x = 879.3 \text{ m}$$

**Answers:**

1)  $\angle B = 22.6^\circ, \angle C = 31.4^\circ, a = 18.9$

2)  $\angle A = 20.8^\circ, \angle B = 24.5^\circ, \angle C = 134.7^\circ$

3) 157.0 km