Name: ______________________

Lesson 8.2 Conjecture and Counter-Examples

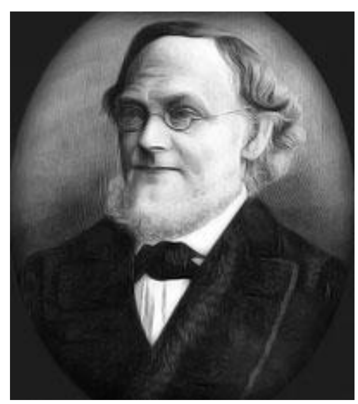
If the same result occurs over and over again, we may conclude that it will always occur. This kind of reasoning is called ______________________ **reasoning.** Inductive reasoning can lead to a ______________________ which is a testable expression that is based on available evidence but is not yet proved. All sciences are based on inductive reasoning (scientific method).

Famous Conjectures

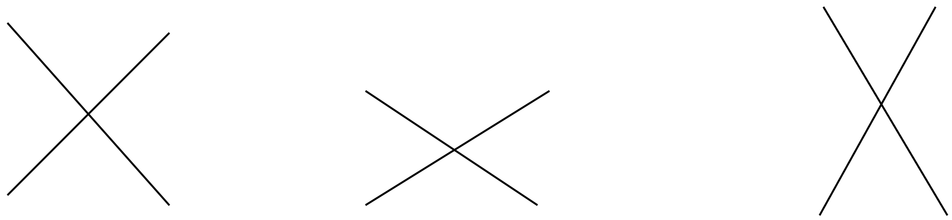
1) Fermat's Last Theorem

$$x^n + y^n = z^n$$

2) Goldbach's Conjecture



Example 1: Make a conjecture about intersecting lines and the angles formed.



Example 2: Use inductive reasoning to make a conjecture about the product of an odd integer and an even integer.

Example 3: Make a conjecture about the sum of two odd numbers.

Assignment

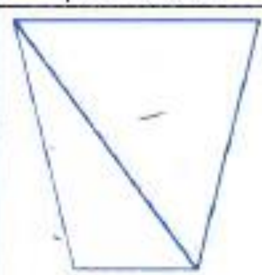
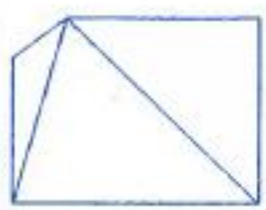
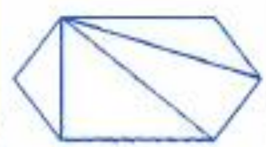
1) Tomas looked at the pattern

17x11= 187	23x11 = 253
41x11 = 451	62x11 = 682

Tomas made this conjecture: When you multiply a two-digit number by 11, the first and last digits of the product are the digits of the original number. Is Tomas's conjecture reasonable? Develop evidence to test the conjecture.

2) Make a conjecture about the sum of two even numbers. Develop evidence to test your conjecture.

3) Use the chart below. Draw the next shape, and make a conjecture about the number of triangles in an *n*-sided shape.

Polygon	quadrilateral	pentagon	hexagon
Fewest Number of Triangles	 2	 3	 4

4) Sonia noticed a pattern when dividing the square of an odd number by 4. Determine the pattern and make a conjecture.

5) Make a conjecture about the sum of one odd integer and one even integer. Test your conjecture with at least three examples.

6) Paula claims that whenever you square an odd integer, the result is an odd number. Is her conjecture reasonable? Justify your decision.

Answer Key

1) Tomas's conjecture is not reasonable. 99(11) = 1089
2) Example: The sum of two even integers is always even. For example. 6+12=18 and 34+72=106
3) Ex. The fewest number of triangles in a polygon is the number of sides subtracted by 2.
4) Ex. The result is always an even number ending with a decimal of .25.
5) Ex. The sum of one odd integer and one even integer is always odd. For example, 3+4=7, -11+44=33, 90+121=211
6) Ex. Paula's conjecture is reasonable. When you multiply an odd digit with an odd digit, the result is odd: 1(1) = 1; 3(3) = 9; 5(5)=25; 7(49); 9(9)=81
Since the ones of a product are the result of a multiplication of two digits, squaring an odd integer will always result in an odd integer.

1.3 Counter Examples

Optical Illusions:

Inductive reasoning can easily lead to ______________________________

We know that inductive reasoning can lead to a conjecture, which may or may not be true. One way a conjecture may be proven false is by a **counterexample.** It takes only one counter example to prove that any conjecture is false.

Example 1: If possible, find a counterexample for each conjecture. If not, write "true".

a) Conjecture: Every mammal has fur.

b) Conjecture: The acute angles in a right triangle are equal.

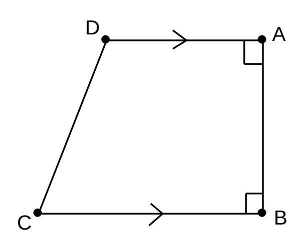
c) Conjecture: A polygon has more sides than diagonals.

d) Conjecture: The square of every even number is even.

e) Conjecture: An even number is any number which is not odd.

Example 2: Three conjectures are given. For which conjectures is this diagram a counterexample?

a) The opposite sides of a parallelogram are equal.



b) A quadrilateral cannot have both a 90° angle and an obtuse angle.

c) Every trapezoid has 2 pairs of equal angles.

Assignment Part 2:

1) Show that each statement is false by providing a counterexample:

   a) A number that is not negative is positive

   b) All prime numbers are odd

   c) All basketball players are tall

   d) The height of a triangle lies inside the triangle

   f) The square root of a number is always less than the number

   g) The sum of two numbers is always greater than the greater of the two numbers

   h) As you travel north, the climate gets colder

2) Jim claims that whenever you multiply two whole numbers, the product is greater than either of the two factors. Do you agree or disagree? Justify your decision.

3) Rachelle claims that the sum of a multiple of 3 and a multiple of 6 must be a multiple of 6. Do you agree or disagree? Justify your decision.

4) Hannah examined these multiples of 9: 18, 45, 63, 27, 81, 108, 216. She claimed that the sum of the digits in any multiple of 9 will add to 9. Do you agree or disagree? Justify you decision.

5) Colin made the following conjecture. If a quadrilateral has two opposite angles that are right angles, the quadrilateral is a rectangle. Do you agree or disagree? Justify your decision.

6) Patrice studies the following table and made this conjecture. The sums of the squares of integers separated by a value of 2 will always be even.

$(-1)^2 + 1^2 = 2$	$2^2 + 4^2 = 20$	$(-3)^2 + (-5)^2 = 34$	$4^2 + 6^2 = 52$	$0^2 + 2^2 = 4$

7) Amy made the following conjecture: When any number is multiplied by itself, the product will be greater than this starting number. For example, in 2x2 = 4, the product 4 is greater than the starting number 2. Meagan disagreed with Amy's conjecture, however, because ½ x ½ = ¼ and ¼ is less than ½. How could Amy's conjecture be improved? Explain.

8) Tim conjectured that all natural numbers can be written as the sum of consecutive natural numbers, based on three examples:

10 = 1+2+3+4
9 = 4+5
12 = 3+4+5
94 = 22+23+24+25

Do you agree or disagree with Tim's conjecture? Justify your decision.

9) Jarrod discovered a number trick in a book he was reading. Choose a number. Double it. Add 6. Double again. Subtract 4. Divide by 4. Subtract 2.

a) Try the trick several times. Make a conjecture about the relation between the number picked and the final result

b) Can you find a counterexample to your conjecture? What does this imply?

Answer Key

1) Answers should vary
a) 0 is a prime number, and is not positive
b) 2 is a prime number that is not odd
c) Muggsy Bogues was an NBA player who was 1.6m (5'3") tall
d) The height of an obtuse triangle can lie outside the triangle
e) not necessarily
f) $\sqrt{0.01} = 0.1$
g) -10 + 5 = -5
h) Try starting at the South Pole

2) Disagree (ex. 1(10)=10)

3) Disagree (ex. 9+12=21)

4) Disagree (ex. 99.9+9=18.18≠9)

5) Disagree (ex. A kite could have angles 90,45,90,135)

6) Patrice's conjecture is reasonable. Integers separated by a value of 2 will both be odd or both be even, and their squares will both be odd of both be even. Adding two even numbers together and adding two odd numbers together result in an even number.

7) Amy's conjecture could be changed to "when any number greater than 1 is multiplied by itself, it will be greater than the starting number"

8) Disagree. The number 2 cannot be written as the sum of consecutive numbers

9) a) The number picked and the final result are the same.
b) I can't find a counter-example. That doesn't mean the conjecture is true, but it does make it more likely.

If the same result occurs over and over again, we may conclude that it will always occur. This kind of reasoning is called **INDUCTIVE reasoning.** Inductive reasoning can lead to a **CONJECTURE** which is a testable expression that is based on available evidence but is not yet proved. All sciences are based on inductive reasoning (scientific method).

Famous Conjectures

1) Fermat's Last Theorem

$$x^n + y^n = z^n$$

Has no non-zero integer solutions for *x*, *y* and *z* when *n* >= 2.

2) Goldbach's Conjecture

Every **even** integer greater than 2 can be represented as a sum of 2 primes.