Unit 1: Logic

1.4 Deductive Reasoning

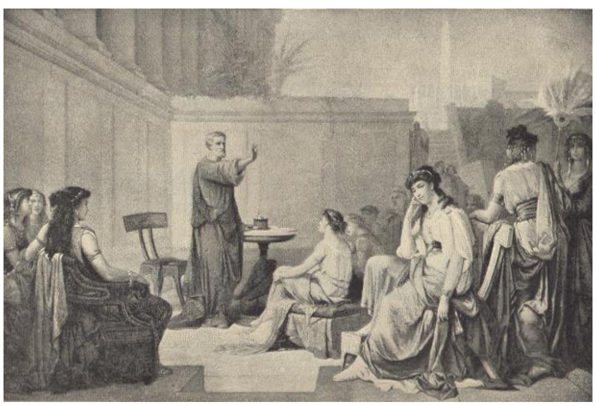
Inductive reasoning is not a proof of anything except for possibilities that you tested. There could always be a counter-example just around the corner.

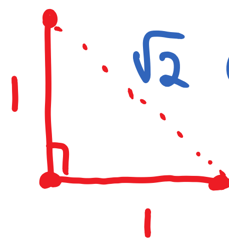
In order to *prove* that something is true, we must use deductive reasoning.

To prove something deductively, it must be proven in the general case (meaning a case that represents any number).

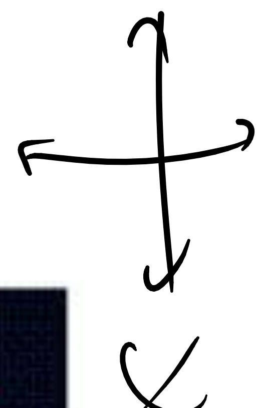
Famous Mathematical Proofs

1) Pythagorus - Irrational Numbers





$\sqrt{2}$ can't be measured precisely.



2) Hilbert's Hotel

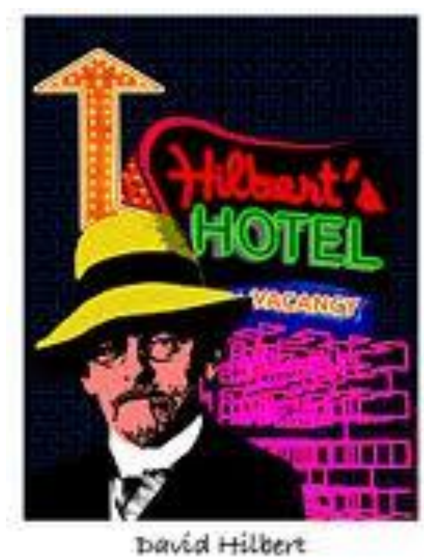
1, 2, 3, ... ∞

Hotel is full

1) fit one more? yes

2) fit infinite more?
ex/ 1, 3, 5, 7 .... ∞

3) fit all rationals? NO!
ex/ 0.1, 3/4, etc..



David Hilbert

3) Gödel's Incompleteness Theorem

No complete system can be completely proven.
(must be assumptions)

ex/ $a+b=b+a$

The chart below shows how to write common math terms in general form in order to make deductive proofs

English	General Form	Restrictions
Any real number	n	$n \in R$
An Integer	n	$n \in I$
2 consecutive integers	$n, n+1$	$n \in I$
An even number ***2 times any Integer is even***	$2n$	$n \in I$
2 consecutive even numbers	$2n, 2n+2$	$n \in I$
An odd number ***an even # plus an odd # is odd***	$2n+1$	$n \in I$
2 consecutive odd numbers	$2n+1, 2n+3$	$n \in I$
Perfect square	n^2	$n \in W$
Consecutive Perfect Squares	$n^2, (n+1)^2$	$n \in W$

Example 1: Use deductive reasoning to prove that the product of an odd integer and an even integer is even.

Inductive

examples

$7 \times 4 = 28$

$5 \times 10 = 50$

$11 \times 6 = 66$

Deductive

any even $= 2n$ any odd $= 2n+1$

product: $2n(2n+1)$

$= 4n^2 + 2n$

$= 2(2n^2 + n)$

Answer always has a factor of 2.

Inductive

Example 2: Choose a number. Add 3. Multiply by 2. Add 4. Divide by 2. Subtract the number you started with. What is the result? Prove why it works.

Pick	8	2	1
Add 3	11	5	4
$\times 2$	22	10	8
$+4$	26	14	12
$\div 2$	13	7	6
$-$orig.	5	5	5

Deductive

Pick	n
$+3$	$n+3$
$\times 2$	$2n+6$
$+4$	$2n+10$
$\div 2$	$n+5$
$-n$	5

Example 3: Use deductive reasoning to prove that the difference between consecutive perfect squares is always an odd number.

Inductive

$3^2 - 2^2 = 5$

$5^2 - 4^2 = 9$

Deductive

$(n+1)^2 - n^2$

$= (n+1)(n+1) - n^2$

$= n^2 + n + n + 1^2 - n^2$

$= 2n + 1$

QED: Answer is always odd.

Assignment:

1) Jim is a barber. Everyone whose hair is cut by Jim gets a good haircut. Austin's hair was cut by Jim. What can you deduce about Austin?

2) Prove that the sum of two even integers is always even.

one even = $2n$ some other even = $2m$

$$2n + 2m = 2(n+m)$$

QED. Answer has factor of 2.

3) Prove that the product of an even integer and an odd integer is always even.

4) Chuck made the conjecture that the sum of any seven consecutive integers in 7 times the median. Prove Chuck's conjecture.

5) Drew created this step-by-step number trick: Choose any number. Multiply by 4. Add 10. Divide by 2. Subtract 5. Divide by 2. Add 3.

a) Show inductively, using three examples, that the result is always 3 more than the chosen number.

b) Prove deductively that the result is always 3 more than the chosen number.

6) Jarrod had a number trick. Choose a number. Double it. Add 6. Double again. Subtract 4. Divide by 4. Subtract 2. Prove that any number you choose will be the final result.

7) Prove that whenever you square an odd integer, the result is odd.

8) Cleo noticed that whenever she determined the difference between the squares of consecutive even numbers of the difference between the squares of consecutive odd numbers, the result was a multiple of 4. Show inductively that this pattern exists. Then prove deductively that it exists.

9) To determine if a number is divisible by 9, add all the digits of the number and determine if the sum is divisible by 9. If it is, then the number is divisible by 9. Prove that the divisibility rule for 9 works for all two-digit and three-digit numbers.

Answer Key

1) Austin got a good haircut
2) $2n + 2m = 2(n + m)$
3) $2m(2n + 1) = 4mn + 2m$
   $2m(2n + 1) = 2(2mn + m)$
4) $(n - 3) + (n - 2) + (n - 1) + n + (n + 1) + (n + 2) + (n + 3) = 7n$
5) $n, 4n, 4n + 10, 2n + 5, 2n, n, n + 3$
6) $n, 2n, 2n + 6, 4n + 12, 4n + 8, n + 2, n$
7) $(2n + 1)^2 = 4n^2 + 2n + 2n + 1$
8) $n^2 - (n - 2)^2 = n^2 - (n - 4n + 4)$
   $n^2 - (n - 2)^2 = n^2 - n^2 + 4n - 4$
   $n^2 - (n - 2)^2 = 4n - 4$
   $n^2 - (n - 2)^2 = 4(n - 1)$
9) Let *ab* represent and two-digit number
ab = 10*a* + *b*
ab = 9*a* + (*a* + *b*)
The number *ab* is divisible by 9 only when (*a* + *b*) is divisible by 9.
Let *abc* represent any three-digit number.
abc = 100*a* + 10*b* + *c*
The number *abc* is divisible by 9 only when (*a* + *b* + *c*) is divisible by 9.

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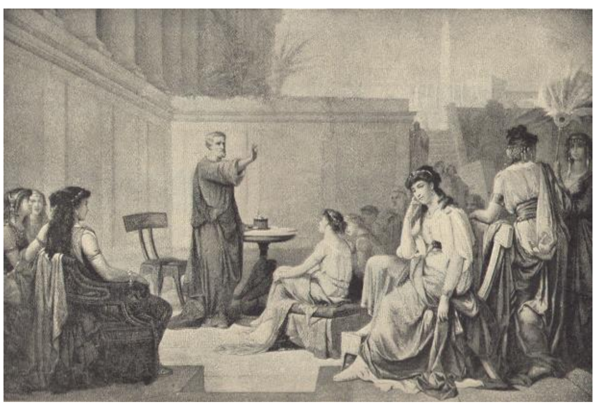
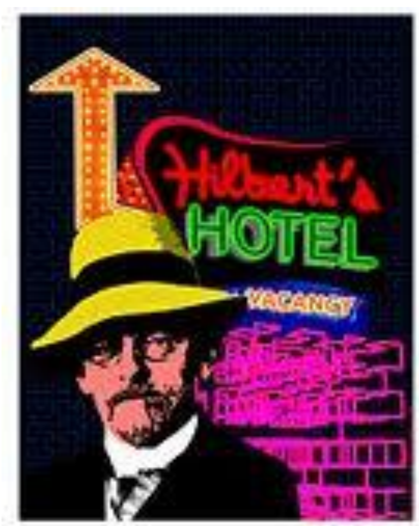


2) Hilbert's Hotel



David Hilbert

3) Gödel's Incompleteness Theorem

Some mathematical statements are true but can't be proven.
(Can't prove everything – always have assumptions)

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