Chapter 1 Review: Logic

Textbook p6-62
Summary: p.34,59-60
Practice Questions p.61-62

Key Concepts: Inductive + Deductive Reasoning

Inductive = Try a bunch of examples and make a conjecture based on the pattern that you find. Or prove that something is false using a counter-example

Deductive = Prove that something is true using logical steps or algebra. Be careful to avoid common errors such as division by 0 or circular logic.

Key Example: The Number Trick

Choose a number
Double it
Add 6
Double again
Subtract 4
Divide by 4
Subtract 2

Inductive

Try a few numbers

Choose a number	2	0	-2
Double it	4	0	-4
Add 6	10	6	2
Double again	20	12	4
Subtract 4	16	8	0
Divide by 4	4	2	0
Subtract 2	2	0	-2

Based on the pattern, I make a conjecture that... the answer is always the same as the original number

Deductive

Let n = any number

Choose a number	n
Double it	$2n$
Add 6	$2n+6$
Double again	$2(2n+6)$ $4n+12$
Subtract 4	$4n+8$
Divide by 4	$n+2$
Subtract 2	n

If I start with any number n, I have proved deductively that... the answer is always the same as the original number.

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Practice #1: Sally noticed a pattern when dividing the square of an odd number by 4. Determine the pattern and make a conjecture. (Inductive Reasoning)

$\frac{1^2}{4}$ $\frac{3^2}{4}$ $\frac{5^2}{4}$ $\frac{7^2}{4}$

0.25, 2.25, 6.25, 12.25

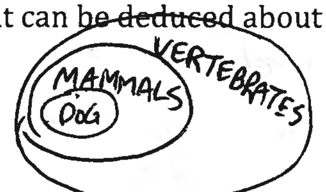
CONJECTURE: Answer always ends with .25.

Practice #2: Jim makes a conjecture that whenever you multiply two whole numbers, the product is always greater that either of the two factors. Do you agree or disagree? (Can you find a counter-example?) 0, 1, 2, 3, ...

$3 \times 4 = 12$ ✓ 12 is greater than 3 and greater than

$5 \times 0 = 0$ COUNTER-EXAMPLE 0 is not greater than 5

Practice #3: All dogs are mammals. All mammals are vertebrates. Shaggy is a dog. What can ~~be deduced~~ about Shaggy?



Shaggy must be a mammal and must be a vertebrate.

Practice #4: Prove that the product of an even integer and an odd integer is always even.

Inductive Reasoning	Deductive Reasoning
$2 \times 3 = 6$ $4 \times -1 = -4$ $-2 \times 5 = -10$ $10 \times 11 = 110$ CAN'T FIND COUNTER-EXAMPLE, THEREFORE CONJECTURE IS TRUE	EVEN NUMBER = $2n$ ODD NUMBER = $2n+1$ PRODUCT $2n(2n+1)$ $= 4n^2 + 2n$ $= \underline{2}(2n+n)$ IF THE PRODUCT ALWAYS HAS A FACTOR OF 2, THEN I HAVE PROVEN DEDUCTIVELY THAT THE CONJECTURE IS ALWAYS TRUE!

Practice #5: Consider the following number-trick:

Choose any number
Multiply by 4.
Add 10.
Divide by 2.
Subtract 5.
Divide by 2.
Add 3.

Use inductive reasoning to make a conjecture about the result and then prove it using deductive reasoning.

Inductive Reasoning	Deductive Reasoning
2 0 -2	Any number = n
$\times 4$ = 8 0 -8	$\times 4$ $4n$
$+10$ = 18 10 2	$+10$ $4n+10$
$\div 2$ = 9 5 1	$\div 2$ $2n+5$
-5 = 4 0 -4	-5 $2n$
$\div 2$ = 2 0 -2	$\div 2$ n
$+3$ = 5 3 1	$+3$ $n+3$
CONJECTURE: ANSWER IS ALWAYS 3 MORE THAN ORIGINAL NUMBER	PROOF: THE ANSWER IS ALWAYS 3 MORE THAN THE ORIGINAL NUMBER.

Practice #6: According to this proof, 2 = 1. Determine the error in reasoning.

$$\text{Let } a = b$$
$$a^2 = ab$$
$$a^2 + a^2 = a^2 + ab$$
$$2a^2 = a^2 + ab$$
$$2a^2 - 2ab = a^2 - ab$$
$$2(a^2 - ab) = 1(a^2 - ab)$$
$$2 = 1$$

$a^2 - ab = 0$
(since $a = b$)

DIVISION BY ZERO ERROR!

Practice #7: Prove that the sum of four consecutive natural numbers is always even. (Deductive Reasoning)

1, 2, 3, ...

$n + (n+1) + (n+2) + (n+3)$ ~~+ (n+4)~~

$= 4n + 6$

$= 2(2n+3)$

SINCE THE ANSWER ALWAYS HAS A FACTOR OF 2, IT MUST BE EVEN!