## Chapter 3&4 Review: Trigonometry

*Textbook p.116-154, 162-200*
*Summary: p.128, 153, 174, 199*
*Practice Questions p.154, 200*

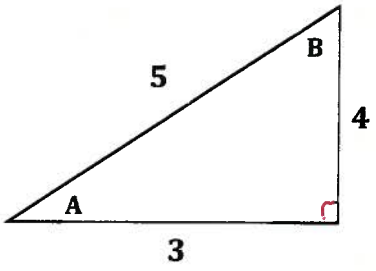
**Key Concepts:** Basic Trig Rations, Cosine Law, Sine Law, The Ambiguous Case

**Basic Trigonometry Ratios**
These ratios only apply to right triangles.

**SOH CAH TOA**

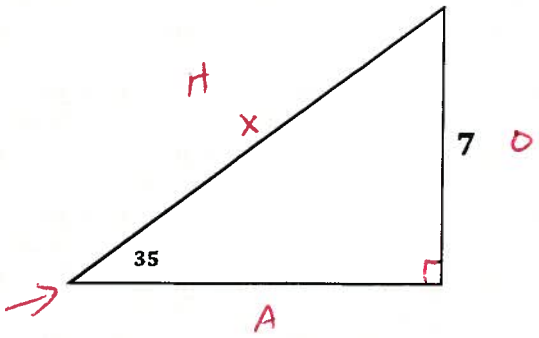
$\sin X = \frac{opposite}{hypotenuse}$ $\cos X = \frac{adjecent}{hypotenuse}$ $\tan X = \frac{opposite}{adjacent}$

**Example:** Find both angles using sine, cosine and tangent.




| To Find Angle A | To Find Angle B |
|---|---|
| $\sin A = \frac{4}{5}$ $\sin^{-1}(\frac{4}{5}) = 53$ | $\sin B = \frac{3}{5}$ $\sin^{-1}(\frac{3}{5}) = 37$ |
| $\cos A = \frac{3}{5}$ $\cos^{-1}(\frac{3}{5}) = 53$ | $\cos B = \frac{4}{5}$ $\cos^{-1}(\frac{4}{5}) = 37$ |
| $\tan A = \frac{4}{5}$ $\tan^{-1}(\frac{4}{5}) = 53$ | $\tan B = \frac{3}{5}$ $\tan^{-1}(\frac{3}{4}) = 37$ |

**Example #2:** Find the hypotenuse




SOH CAH TOA

$\sin 35 = \frac{7}{x}$

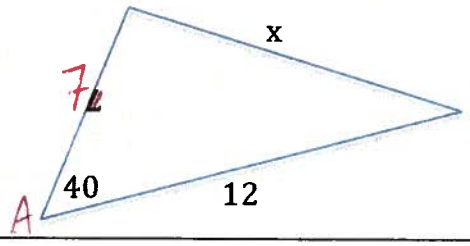
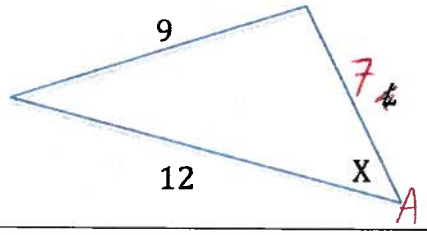
$x = \frac{7}{\sin 35}$

$x = 12.2$

**Cosine Law**

For non-right triangles where you are given SAS or SSS.

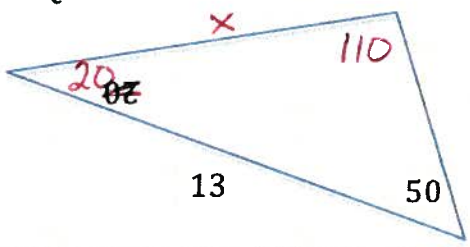
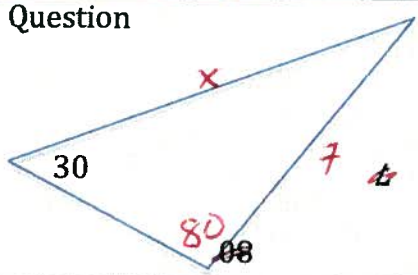
$$a^2 = b^2 + c^2 - 2bc\,cosA$$

| Example #1 | Example #2 |
|---|---|
| SAS Question   | SSS Question   |
| Solution $x^2 = 7^2 + 12^2 - 2(7)(12)\cos 40$ $x^2 = 64.3$ $x = 8.0$ | Solution $9^2 = 7^2 + 12^2 - 2(7)(12)\cos X$ $\frac{9^2 - 7^2 - 12^2}{-2(7)(12)} = \cos X = .667$ $X = 48.2°$ |

**Sine Law**

For non-right triangles where you are given ASA or AAS.

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

| Example #1 | Example #2 |
|---|---|
| ASA Question   | AAS Question   |
| Solution $\frac{13}{\sin 110} = \frac{x}{\sin 50}$ $x = 10.6$ | Solution $\frac{7}{\sin 30} = \frac{x}{\sin 80}$ $x = 13.8$ |

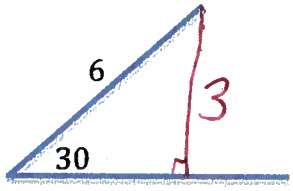
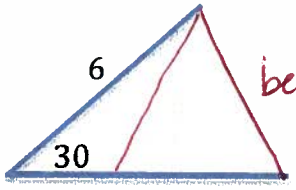
**The Ambiguous Case of the Sine Law**

For triangles where you are given ASS, there may be no solutions, one solution or two solutions.

To determine which case you have, compare the second given side to the height of the triangle.

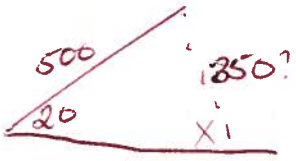
**Example:** One angle is 30, the adjacent side is 6, and the next side is.... Height: $\sin 30 = \frac{h}{6}$

$h = 3$

| **Case #1 (no solution)** | **Case #2 (one solution - right)** |
|---|---|
|   |   |
| **Case #3 (one solution - obtuse)** | **Case #4 (two solutions - ambiguous)** |
|   |   |

**Key Example:** A landowner says that his property is triangular, with one side 500 m long and another side 350 m long. The opposite angle to the 350 m side measures 20°. Determine the length of the third side, to the nearest metre. Show your work.

STEP #1 DRAW




$\sin 20 = \frac{h}{500}$

$h = 171$

TWO SOLUTIONS

STEP #2 SOLVE

$\frac{\sin 20}{350} = \frac{\sin X}{500}$

$\sin X = .4886$

$X = 29.2°$ OR $150.8°$

$= 29°; 151°$

STEP #3 ANSWER

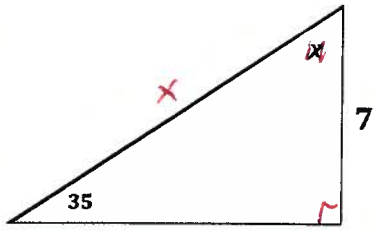
TRIANGLE 1

TRIANGLE 2

SOH

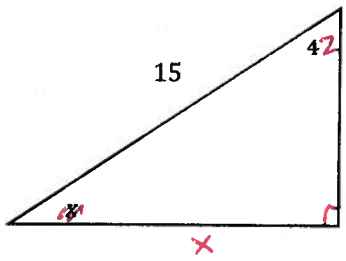
## Chapter 3&4 Review: Trigonometry

**Practice #1:** Find the unknown length or angle for each triangle ($x$).




$\sin 35 = \frac{7}{x} \rightarrow x = \frac{7}{\sin 35}$

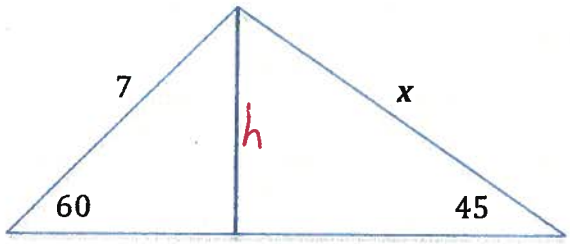
$x = 12.2$




$\sin 42 = \frac{x}{15}$

$x = 15 \sin 42$

$x = 10.0$



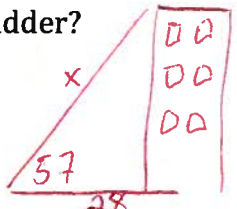

$\sin 60 = \frac{h}{7}$

$h = 7 \sin 60 = 6.06$

$\sin 45 = \frac{h}{x} \rightarrow x = \frac{6.06}{\sin 45}$

$x = 8.6$

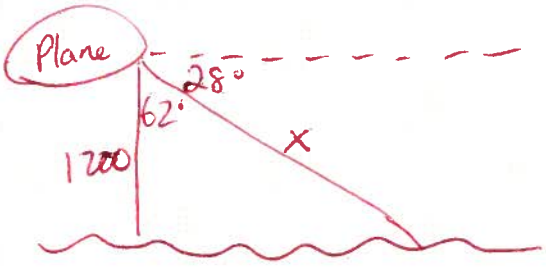
**Practice #2:** A fireman rests his ladder against a building, making a 57° angle with the ground. The bottom of the ladder is 28 feet from the base of the building. How long is the ladder?




$\cos 57 = \frac{28}{x} \rightarrow x = \frac{28}{\cos 57}$

$x = 51.4$ ft

**Practice #3:** The pilot of an airplane in flight looks down at a point on the ground that is some distance away. The angle of depression is 28°, and the plane's altitude is 1200 meters. What is the distance from the pilot to the point on the ground?

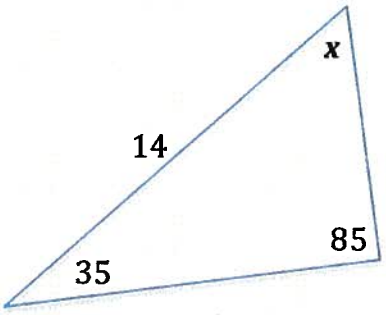



$\cos 62 = \frac{1200}{x}$

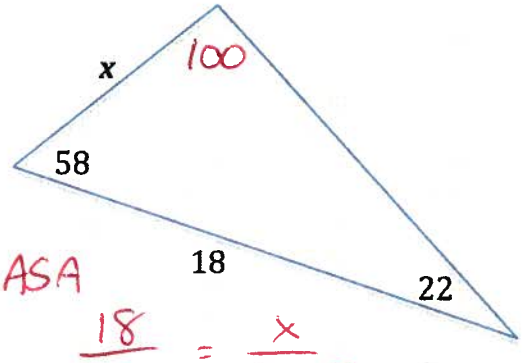
$x = \frac{1200}{\cos 62}$

$x = 2556$ metres

**Practice #4:** Find the unknown length or angle for each triangle ($x$).



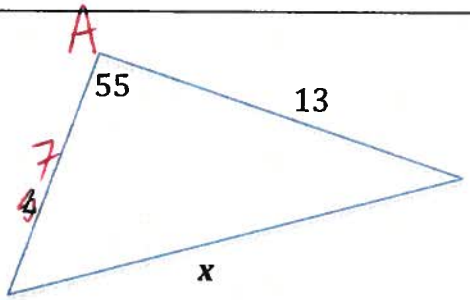

$x = 60°$
(angles add to 180°)




ASA

$\frac{18}{\sin 100} = \frac{x}{\sin 22}$

$x = 6.8$

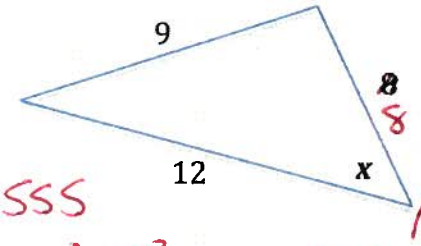



SAS

$x^2 = 7^2 + 13^2 - 2(7)(13)\cos 55$

$x^2 = 113.6$

$x = 10.7$

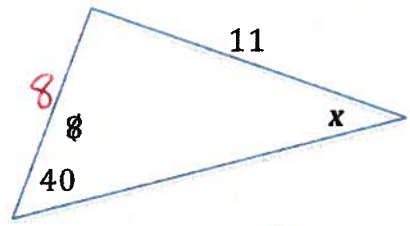



SSS

$9^2 = 8^2 + 12^2 - 2(8)(12)\cos X$

$\frac{9^2 - 8^2 - 12^2}{-2(8)(12)} = \cos X = .661$

$X = 48.6°$




ASS

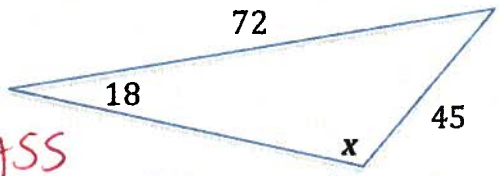
$\frac{\sin 40}{11} = \frac{\sin X}{8}$

$.467 = \sin X$

$X = 27.9°$

OR 152.1? $180 - 27.9$

NO, because $11 > 8$




ASS

$\frac{\sin 18}{45} = \frac{\sin X}{72}$

$\sin X = .4944$

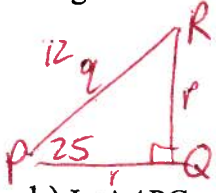
$X = 29.6°$

OR $X = 150.4°$ $180 - 29.6$

TWO ANSWERS
(its ambiguous)

**Practice #5:** Solve each of the following triangles (ie. draw the triangle and label ALL of the unknown angles and lengths)

a) In a right triangle $\Delta PQR$, the hypotenuse, $q$, is 12 m long and $\angle P = 25°$. Determine the length of sides $p$ and $r$ to the nearest tenth of a metre.




$\sin 25 = \frac{p}{12}$

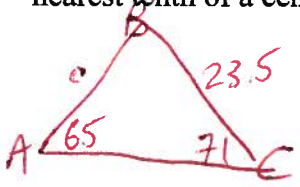
$p = 12\sin 25 = 5.1$

$\cos 25 = \frac{r}{12}$

$r = 12\cos 25$

$r = 10.9$

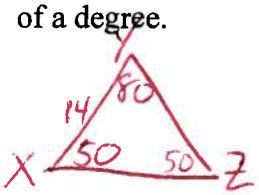
b) In $\Delta ABC$, $\angle A = 65°$, $a = 23.5$ cm, and $\angle C = 71°$. Determine the length of side $c$ to the nearest tenth of a centimetre.




$\frac{23.5}{\sin 65} = \frac{c}{\sin 71}$

$c = 24.5$

c) In $\Delta XYZ$, $\angle X = 50°$, $\angle Y = 80°$, and $z = 14$ cm,. Determine angle Z, to the nearest tenth of a degree.



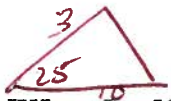

50° (angles add to 180°)

**Practice #6:** For each description below determine if there are zero, one, or two possible triangles.

a. In ΔDEF, d = 5cm, e = 3cm, f = 9cm .

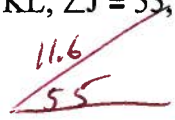
SSS → one solution

b. In ΔABC, ∠A = 25 ,b = 3m, c =10m .




SAS → one solution

c. In ΔJKL, ∠J = 55, j =10.4km, k =11.6km.




$\sin 55 = \frac{h}{11.6}$

$h = 9.5$

$9.5 < 10.4 < 11.6$

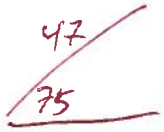
between

→ TWO SOLUTIONS

d. In ΔPQR, ∠P =17, ∠Q =110, r = 26mm.


ASA → one solution

e. In ΔFUN, ∠F = 75, f = 25cm, n = 47cm.




$\sin 75 = \frac{h}{47}$

$h = 45.4$

less than h

NO SOLUTION!

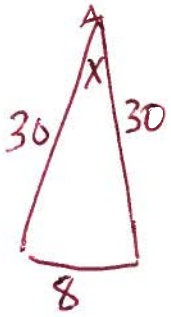
**Practice #7:** Write another sine ratio that is equivalent to sin 44°.

sin 136

**Practice #8:** Determine two angles between 0° and 180° that have the sine ratio 0.8480

58°, 122°

**Practice #9:** At Science World, there is a giant pendulum on display. The line is 30 feet long, and when the pendulum swings from side to side, the horizontal distance it travels is 8 ft. Determine the angle through which the pendulum swings. Round your answer to the nearest inch.




SSS

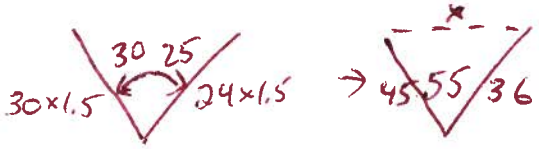
$8^2 = 30^2 + 30^2 - 2(30)(30)\cos X$

$\frac{8^2 - 30^2 - 30^2}{-2(30)(30)} = \cos X$

$\cos X = .9644$

$X = 15.3°$

**Practice #10:** Two boats leave the dock at the same time. One is going an average of 30 km/h in the direction N30W, and the other is going an average of 24 km/h in the direction N25E. How far apart are the boats after 1.5 hours?

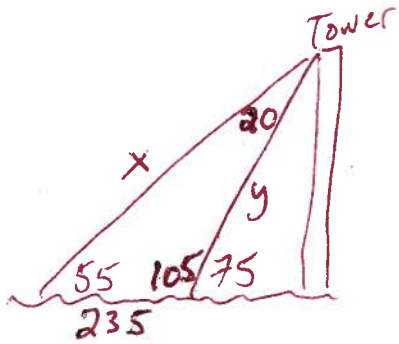

SAS

$x^2 = 45^2 + 36^2 - 2(45)(36)\cos$

$x^2 = 1462.6$

$x = 38.2$ km

**Practice #11:** A radio tower is supported by two wires on opposite sides. On the ground, the ends of the wire are 235 m apart. One wire makes a 75° angle with the ground. The other makes a 55° angle with the ground. Draw a diagram of the situation. Then, determine the length of each wire to the nearest metre.




ASA

$\frac{x}{\sin 105} = \frac{235}{\sin 20}$

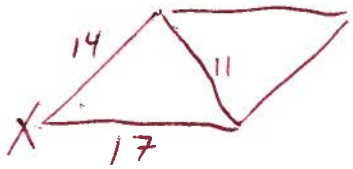
$x = 663.7$

$\frac{y}{\sin 55} = \frac{235}{\sin 20}$

$y = 562.8$

664 m + 563 m

**Practice #12:** In a parallelogram, two adjacent sides measure 17 cm and 14 cm. The shorter diagonal is 11 cm. Determine, to the nearest degree, the measures of the larger angles in the parallelogram.




larger + smaller = 180

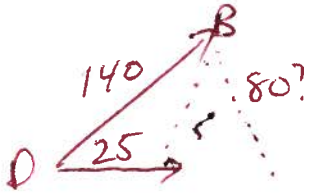
Answer = 180 − X

$11^2 = 14^2 + 17^2 - 2(14)(17)\cos X$

$\frac{11^2 - 14^2 - 17^2}{-2(14)(17)} = \cos X = 40.1°$

180 − 40.1 = 140°

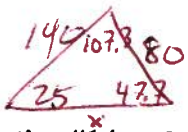
**Practice #13:** A canoeist leaves the dock and paddles toward a buoy 140 m away. After reaching the buoy, she changes directions and paddles another 80 m. From the dock, the angle between the buoy and the canoeist's current position measures 25°. How far is the canoeist from the dock? Give two possible answers. Show your work.




$\sin 25 = \frac{h}{140}$ $h = 59$ TWO SOLUTIONS

$\frac{\sin 25}{80} = \frac{\sin X}{140}$ $X = 47.7°$ OR $132.3°$ $(180 - 47.7)$

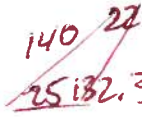
TRIANGLE #1




$\frac{x}{\sin 107.3} = \frac{80}{\sin 25}$

$x = 180.7$ m

TRIANGLE #2

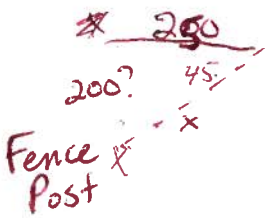



$\frac{x}{\sin 22.7} = \frac{80}{\sin 25}$

$x = 73.1$ m

**Practice #14:** A farmer finishes repairing a fence post and then walks 200 yd through his corn field. He turns and walks another 250 yd east, until he can see the fence post directly southwest of him. He realizes that he left some of his tools at the fence post and heads directly back to it. How far does he need to walk, to the nearest metre?

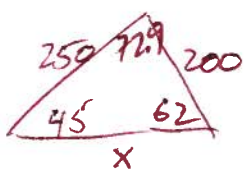
N E S




$\sin 45 = \frac{h}{250}$ $h = 177$ TWO SOLUTIONS

$\frac{\sin 45}{200} = \frac{\sin X}{250}$ $X = 62.1°$ OR $117.9°$ $(180 - 62.1)$

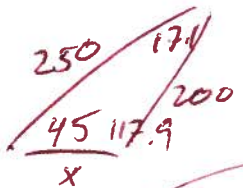
TRIANGLE #1




$\frac{x}{\sin 72.9} = \frac{200}{\sin 45}$

$x = 270$ m

TRIANGLE #2




$\frac{x}{\sin 17.1} = \frac{200}{\sin 45}$

$x = 83.4$ m