Chapter 6 Review: Linear Inequalities

Textbook p.294-350
Summary: p.321-322, p.348
Practice Questions p.323, p.349-350

Key Concepts: Graphing Linear Inequalities, Systems of Inequalities, Set Notation, Objective Function, Optimization

Inequalities Vocabulary

A number less than 3: $x<3$
A number greater than -2: $x>-2$
A number not larger than -5: $x\leq-5$
A number greater than or equal to 7: $x\geq 7$
A number between 0 and 5: $0<x<5$
A number less than 5 or greater than 10: $5>x>10$
A positive number: $x>0$

$<$ OR $>$ DOTTED
$\leq$ OR $\geq$ SOLID

Graphing Linear Inequalities

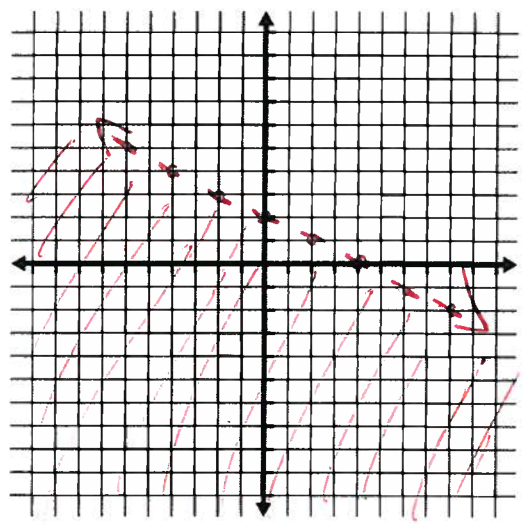
Graph $x+2y<4$

STEP #1 – Solve for y $2y<-x+4$
$y<-\frac{1}{2}x+2$

STEP #2 – Graph using slope + y-intercept
slope $=-\frac{1}{2}$ y-int $=2$

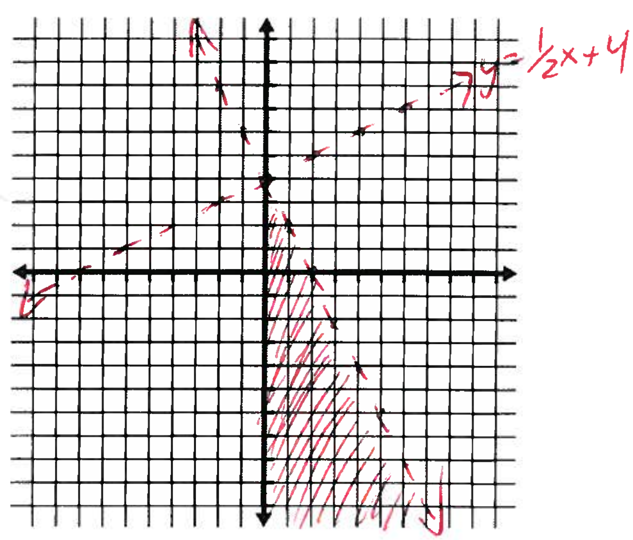
STEP #3 – Test Point
$(0,0)$ $0+2(0)<4$
$0<4$ ✓

STEP #4 – Shade



Graphing a System of Linear Inequalities

$$\text{Graph: } \begin{aligned} &x\geq 0 \\ &y<\frac{1}{2}x+4 \\ &y<-2x+4 \end{aligned}$$



Solution is where ALL are true
(can use regular OR reverse shading)

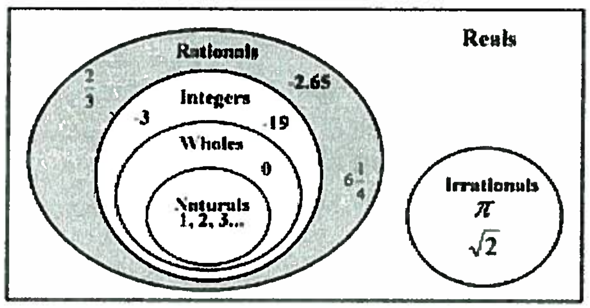
Set Notation

$$\{(x, y) | y > -2x + 1, x \in R, y \in R\}$$

"The <u>set</u> of all (x,y) coordinates <u>such that</u> $y > -2x + 1$ x is an <u>element</u> of the <u>real</u> numbers and y is an <u>element</u> of the <u>real</u> numbers."

Domain: All of the possible x-values (left to right)
Range: All of the possible y-values (down to up)

Types of Numbers:



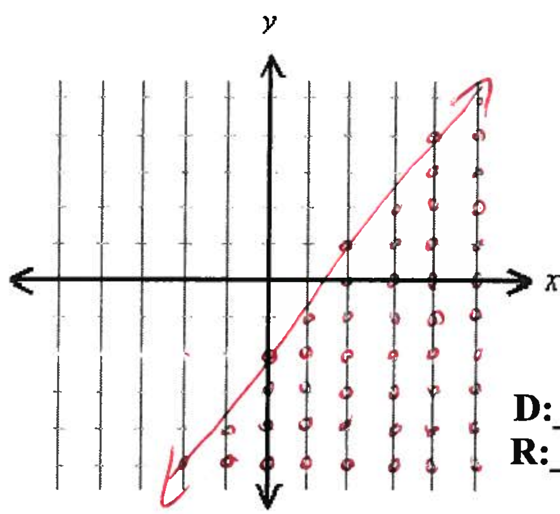
N - Naturals
W - Whole
I - Integers] STIPPLES
R - Reals] SHADING

Example Graph the solution for $\{(x, y) | 2y + 4 \leq 3x, x \in I, y \in I\}$

$2y + 4 \leq 3x$

$2y \leq 3x - 4$

$y \leq \frac{3}{2}x - 2$



D: <u>$x \in I$</u>
R: <u>$y \in I$</u>

Optimization Vocabulary

Constraint = Inequality or Restriction

Feasible Region = The Solution Space

Objective Function = An equation for whatever you are trying to maximize/minimize

Maximize/Minimize = Finding the value using objective function

Optimize = Using a system of inequalities to maximize/minimize a value.

Key Example A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

STEP #1: Define Variables

r = racing cars
s = SUV's

STEP #2: Write Constraints

$r \le 40 \quad s \le 60$

$r + s \ge 70$

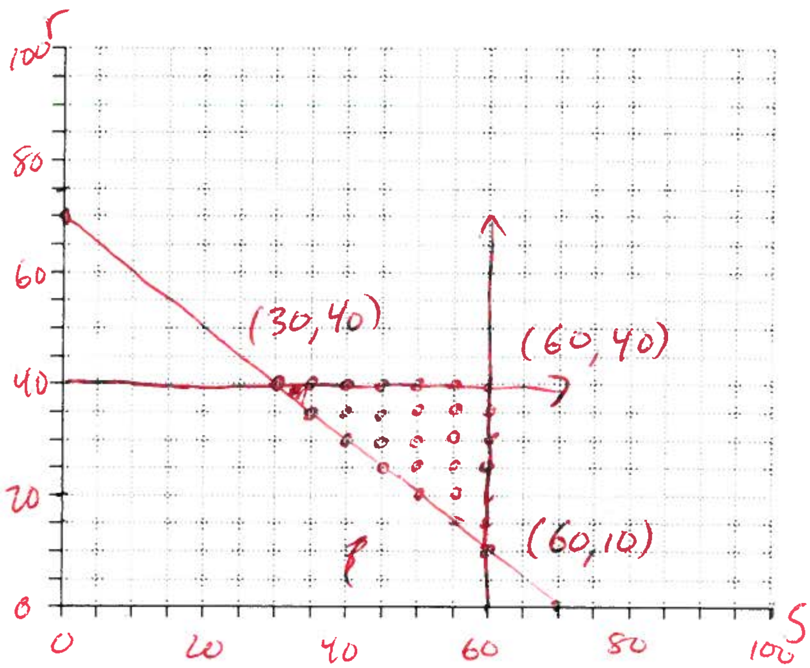
$\rightarrow r \ge -s + 70$

STEP #3: Note Restrictions

~~$r, s \ge 0$~~

$r, s \in W$

STEP #4: Graph



STEP #5: Label Corners

STEP #6: Objective Function

$C = 8r + 12s$

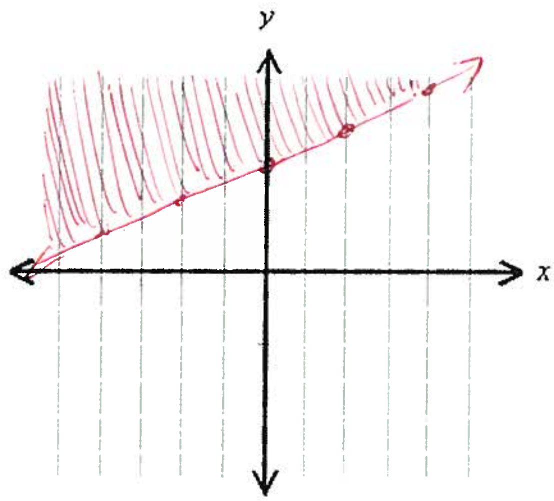
STEP #7: Maximum/Minimum

Point	$C = 8r + 12s$	Cost			
(30, 40)	8(40) + 12(30)	\$680	MINIMUM	30 SUVs	40 cars
(60, 40)	8(40) + 12(60)	\$1040	MAXIMUM	60 SUVs	40 cars
(60, 10)	8(10) + 12(60)	\$800			

Chapter 6 Review: Inequalities

Practice #1: Graph each of the following.

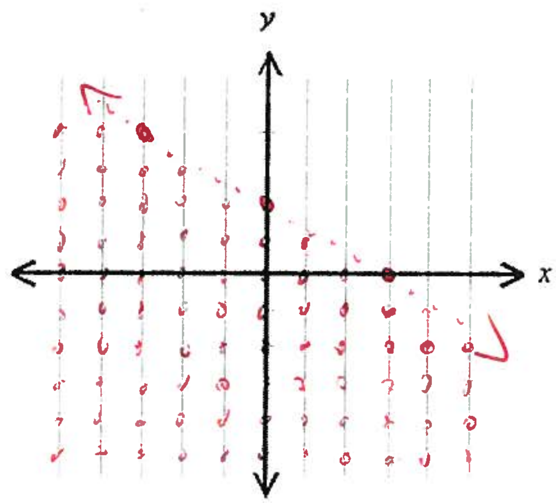
a) $\left\{(x,y)\,\middle|\, y \geq \frac{1}{2}x + 3, x \in R, y \in R\right\}$



b) $\{(x,y) \mid -3y + 6 < 2x, x \in I, y \in I\}$

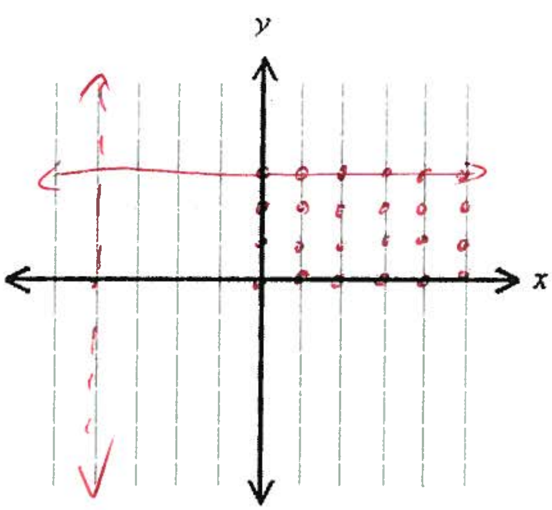
$-3y < 2x - 6$

$y < -\frac{2}{3}x + 2$



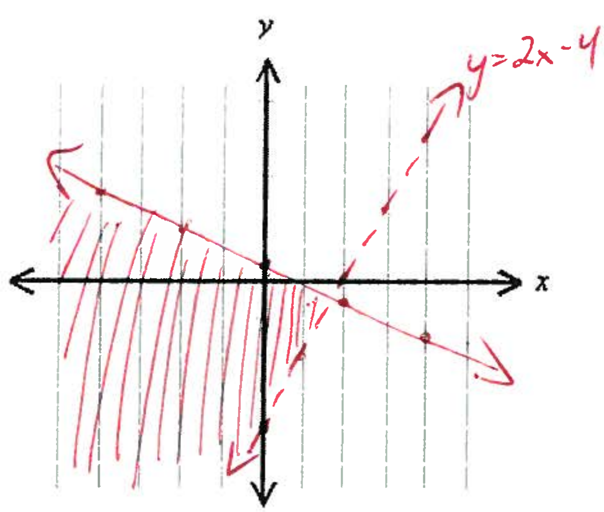
c) $\{(x,y) \mid y \leq 3, x > -4, x \in W, y \in W\}$

No negatives



d) $\{(x,y) \mid y > 2x - 4, 2y \leq -x + 1, x \in R, y \in R\}$

$y \leq -\frac{1}{2}x + \frac{1}{2}$



Practice #2: A student council is ordering signs for the winter dance. Signs can be made in letter size or poster size.

- No more than 30 of each size are wanted.
- No more than 50 signs are needed altogether.
- Letter-size signs cost \$8.75 each, and poster-size signs cost \$14.50 each.

Let *l* represent the number of letter-size signs.
Let *p* represent the number of poster-size signs.
Write the objective function to determine the combination of the two sizes of signs that would result in the lowest cost to the council.

$C = 8.75l + 14.50p$

Practice #3: A system of linear inequalities has vertices at (–2, –6), (1, –4), (4, –6), and (0, –10). Which point represent the maximum value of the objective function *W* = 0.5*y* + 3*x* – 10?

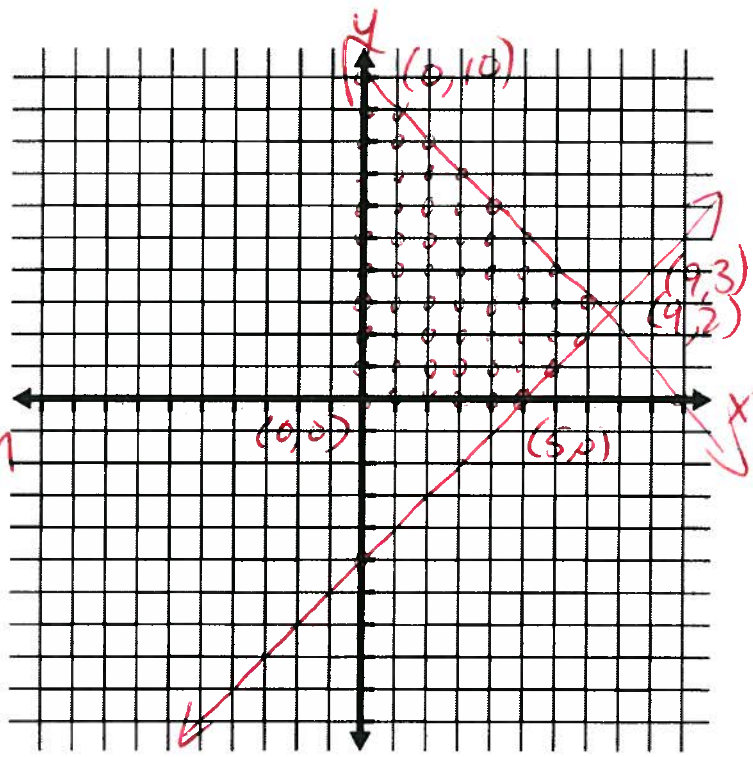
Point	Objective Function	Value
(-2,-6)	0.5(-6)+3(-2)-10 =	-19
(1,-4)	0.5(-4)+3(1)-10 =	-9
(4,-6)	0.5(-6)+3(4)-10 =	-1 ← MAXIMUM at (4,-6)
(0,-10)	0.5(-10)+3(0)-10 =	-15

Practice #4: The following model represents an optimization problem. Determine the maximum solution and show your work.

Restrictions:
$x \in W$
$y \in W$

Constraints:
$x > 0$
$y > 0$
$x \leq y + 5$ $\quad y \geq x - 5$
$x + y \leq 10$ $\quad y \leq -x + 10$

Objective function:
$A = x + 2y$



(0,10)	0+2(10) = 20	MAXIMUM
(9,3)	9+2(3) = 15	
(9,2)	9+2(2) = 13	
(5,0)	5+2(0) = 5	
(0,0)	0+2(0) = 0	

Practice #5: A publisher makes romance and adventure novels. Romance novels sell for $10 and adventure novels for $8. The publishers noticed that each month they always sell between 500 and 800 romance novels and that the number of adventure novels sold is never more than double the number of romance novels sold. What are the maximum and minimum profits for a month?

VARIABLES

r = romance

a = adventure

CONSTRAINTS

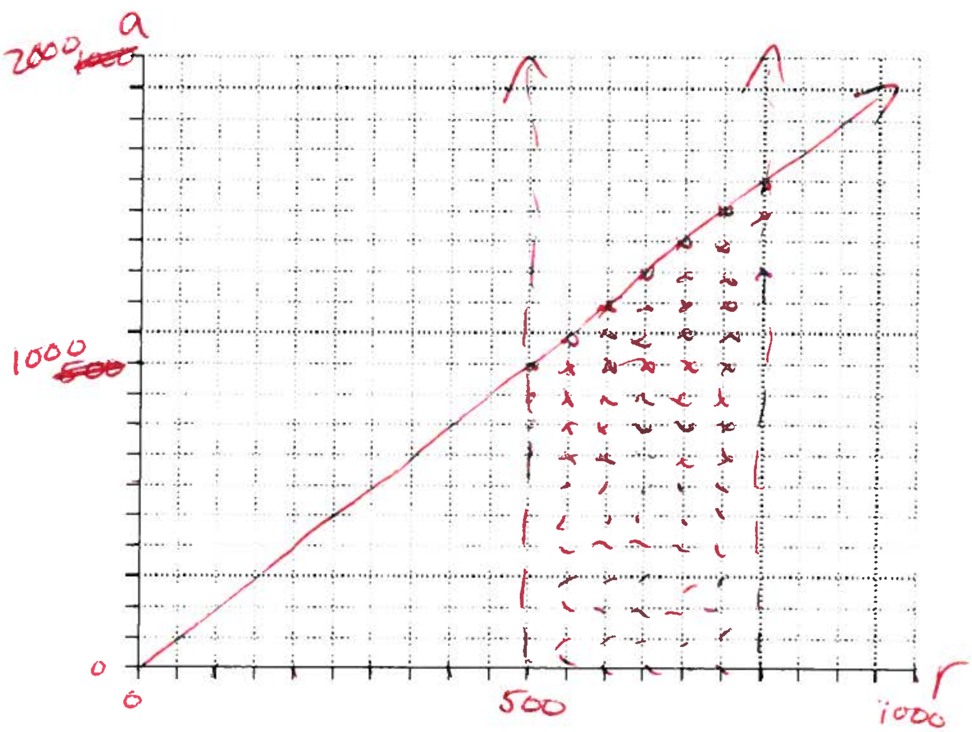
$500 < r < 800$

$a \leq 2r$

$r, a \in W$

OBJECTIVE FUNCTION

$P = 10r + 8a$



CORNERS

(500, 0)	10(500) + 8(0)	$5000 MINIMUM
(500, 1000)	10(500) + 8(1000)	= $13 000
(800, 0)	10(800) + 8(0)	= $8000
(800, 1500)	10(800) + 8(1500)	= $20 000 MAXIMUM

Practice #6: A test is made up of multiple-choice and open-ended questions.

- It takes 3 minutes to do a multiple-choice question and 6 minutes for an open-ended question.
- The total time for the test is 60 minutes
- You may answer no more than 20 questions.
- Multiple choice questions are worth 1 mark and open-ended questions are worth 3 marks

Define the relevant variables, write an inequality for each constraint in the question, draw the feasible region, and create an objective function to maximize your score on the test.

VARIABLES

m = multiple-choice

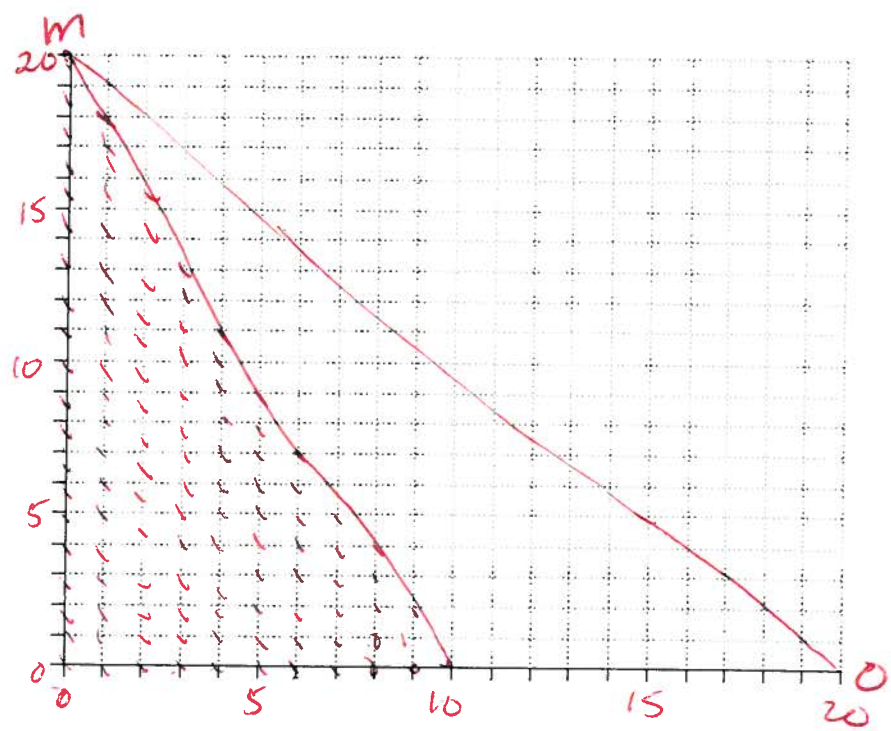
o = open-ended

CONSTRAINTS

$3m + 6o \leq 60$

$m \leq -2o + 20$

$m + o \leq 20$



$m, o \in W$

OBJECTIVE $M = m + 3o$

CORNERS

(0,0) $0 + 3(0) = 0$

(0,20) $20 + 3(0) = 20$

(10,0) $0 + 3(10) = 30$ ← MAX → ANSWER 10 open-ended questions!