Chapter 7 Review: Quadratics

Textbook p.358-444
Summary: p.396-397, p.441-442
Practice Questions p.398,p.443-444

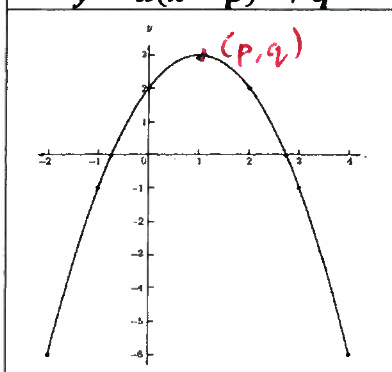
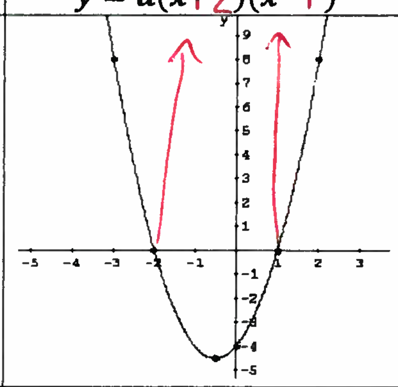
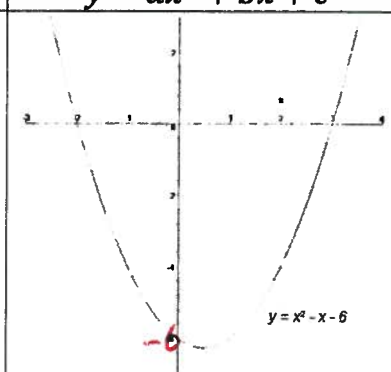
Key Concepts: Quadratic Analysis, Different Forms of Quadratics, Solving Quadratics, Factoring, Quadratic Formula, Modeling with Quadratics.

Definition: A quadratic is any equation with an x^2 in it. (and no $x^{3+}, x^{-n}, \frac{1}{x}, \sqrt{x}, n^x$).
The graph of a quadratic equation is in the shape of a parabola.

Quadratic Analysis

1) The vertex is where the parabola changes direction (either a **maximum** or a **minimum**)
2) The Axis of symmetry for a parabola is a vertical line that goes through the vertex (about which the graph is symmetrical)
3) The **domain** for all quadratics (that aren't restricted in some way by a word problem) is all real numbers
4) The **range** for a quadratic is always restricted by either the maximum or minimum point.
5) Intercepts occur when the graph crosses the x or y axis.
6) The x-intercepts always correspond to the factors of the equation, and are also called solutions, roots, or zeros.

Different Forms of the quadratic

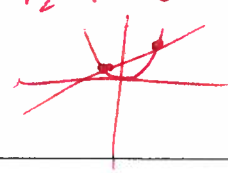
Vertex Form $y = a(x-p)^2 + q$	**Factored Form** $y = a(x+2)(x-1)$	**Standard Form** $y = ax^2 + bx + c$
		
Vertex = (p, q) **p = horizontal shift** **q = vertical shift** **a = stretch factor**	**Factors = x-intercepts**	**c = y-intercept**

You should also be able to find points on a quadratic graph using your calculator. Input the equation in Y=, and use the **CALC** menu.

Solving Quadratics

Solving = finding solutions/roots/zeros/x-intercepts (all the same thing)

Example: Solve $x^2 = x + 6$

Method #1: Factor	**Method #2:** Formula
$x^2 - x - 6 = 0$ $(x+2)(x-3) = 0$ $x = -2, 3$	$a=1$ $b=-1$ $c=-6$ $x^2 - x - 6 = 0$ $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$ $x = \dfrac{1 \pm \sqrt{25}}{2} \rightarrow \dfrac{1 \pm 5}{2}$ 3 OR -2
Method #3: Graph the Zeros	**Method #4:** Graph the Intersections
$Y_1 = x^2 - x - 6$  CALC zero $x = -2, 3$	$Y_1 = x^2$ $Y_2 = x + 6$  CALC intersect $x = -2, 3$

Quadratic Modeling

Many real-life problems can be modeled using quadratic equations. This involves using information from a question to write a quadratic.

Example #1: A parabola has a y-intercept of -4 and a vertex at (3,-7). Write the equation for this quadratic

$y = a(x-p)^2 + q$

vertex = $(3,-7)$ point $(0,-4)$
$p \quad q$

$-4 = a(0-3)^2 - 7$

$3 = a(-3)^2$

$3 = 9a \rightarrow a = \frac{1}{3}$

$\boxed{y = \frac{1}{3}(x-3)^2 - 7}$

Example #2: A parabola has x-intercepts of -3 and 5 and goes through the point (2,15). Write the equation for this quadratic

$y = a(x+3)(x-5)$

$15 = a(2+3)(2-5)$

$15 = a(5)(-3)$

$15 = -15a$

$a = -1$

$\boxed{y = -(x+3)(x-5)}$

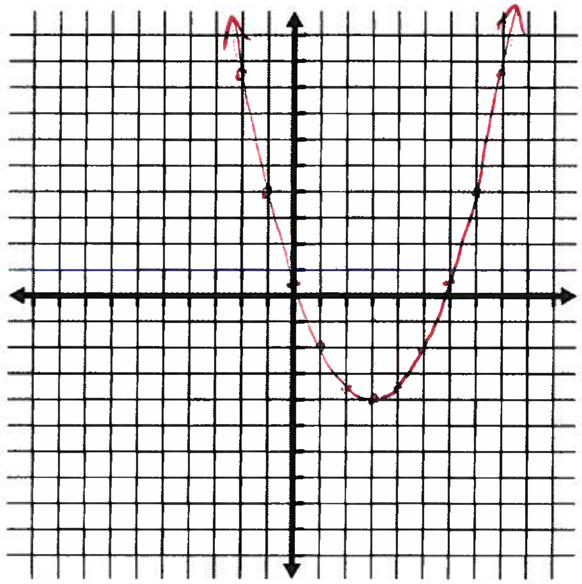
Chapter 7 Review: Quadratics

Practice #1: Graph the equation $y = \frac{1}{2}(x-3)^2 - 4$ without a calculator.

p = 3 q = -4 a = 1/2

vertex = (3, -4)

max or min? MIN



1, 3, 5
.5, 1.5, 2.5

x	y
3	-4
4	-3.5
5	-2
6	.5
2	-3.5
1	-2
0	.5

Domain: all reals
Range: $y \geq -4$

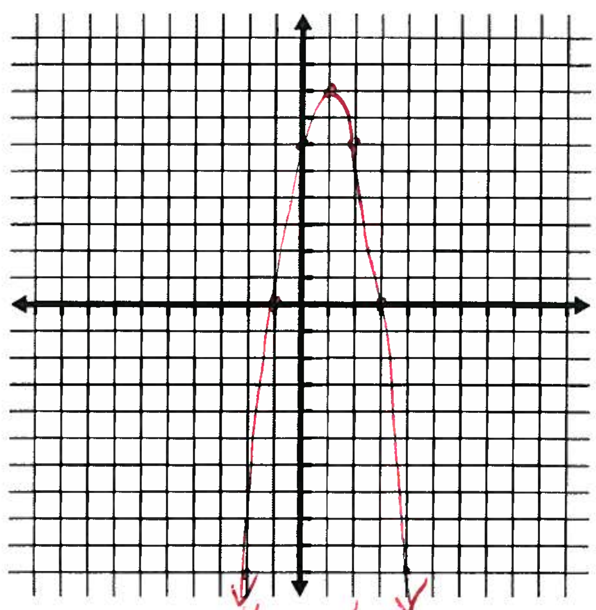
Practice #2: Graph the equation $y = -2(x-3)(x+1)$ without a calculator.

x-ints = 3, -1

a = -2

max or min? MAX

$\frac{3+-1}{2} =$ Axis of Symmetry? $x = 1$



-1, 3, 5
-2, -6, -10

x	y
1	8 $-2(1-3)(1+1)$
2	6
3	0
0	6
-1	0

Domain: all reals
Range: $y \leq 8$

Practice #3: Graph the equation $y = -\frac{1}{2}x(x-4)$ without a calculator.

x-ints = $0, 4$

a = $-1/2$

max or min? MAX

$\frac{0+4}{2}$

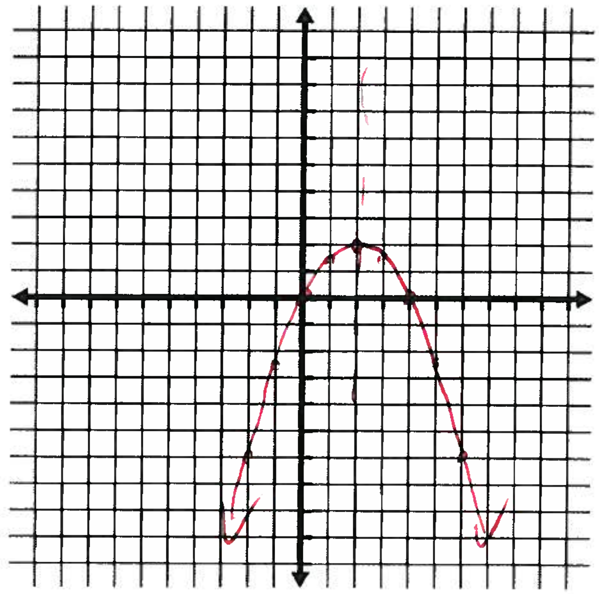
Axis of Symmetry? 2

1, 3, 5

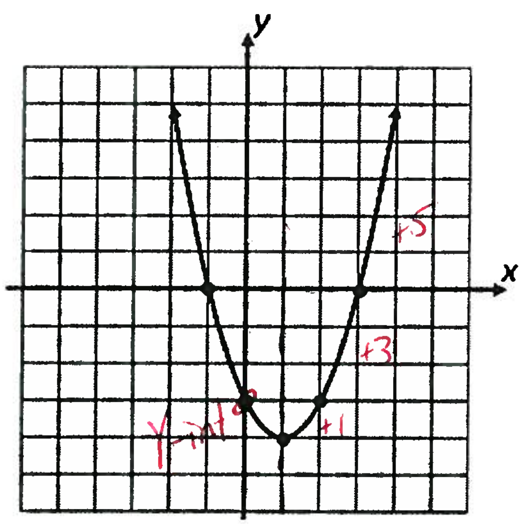
.5, 1.5, 2.5

Domain: all reals

Range: $y \leq 2$



Practice #4: Analyze the graph and write an equation to match:



Vertex: $(1, -4)$

Axis of Symmetry: $x = 1$

Y-Intercept: -3

X-Intercepts: $-1, 3$

Domain: all reals

Range: $y \geq -4$

Pattern: $1, 3, 5, \ldots$

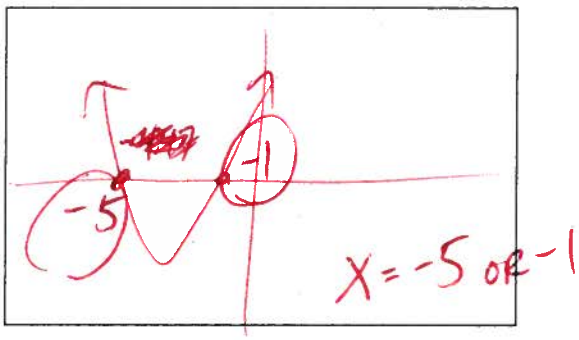
Equation: $y = (x+1)(x-3)$

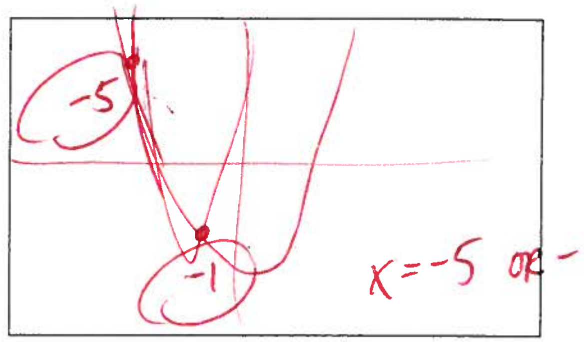
OR

$y = (x-1)^2 - 4$

Practice #5: Solve by graphing: $3x^2 + 4x - 2 = 2x^2 - 2x - 7$

Method #1 – Zeros	**Method #2 – Intersections**
$3x^2+4x-2=2x^2-2x-7$ $x^2+6x+5=0$	$Y_1 = 3x^2+4x-2$ $Y_2 = 2x^2-2x-7$





Practice #6: Solve by factoring

a) $x^2 - 64 = 0$ $\sqrt{x^2} = \sqrt{64}$ $(x+8)(x-8)=0$ $x = \pm 8$ $x = -8, 8$	b) $2x^2 - 16x = -24$ $2x^2-16x+24=0$ $2(x^2-8x+12)=0$ $2(x-2)(x-6)=0$ $x = 2, 6$
c) $x^2 - 2x - 8 = 0$ $(x+2)(x-4)=0$ $x = -2, 4$	d) $2x^2 + 5x - 12 = 0$ $(2x-3)\left(\frac{2x+8}{2}\right)=0$ $(2x-3)(x+4)=0$ $x = \frac{3}{2}, -4$

Practice #7: Use the quadratic formula to solve. *Show all answers in EXACT form and SIMPLIFY if possible*

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a) $x^2 + 5x - 3 = 0$ $x = \dfrac{-5 \pm \sqrt{25 - 4(1)(-3)}}{2(1)}$ $x = \dfrac{-5 \pm \sqrt{37}}{2}$	b) $x^2 - 5x = 2$ $x^2 - 5x - 2 = 0$ $x = \dfrac{5 \pm \sqrt{25 - 4(1)(-2)}}{2(1)}$ $x = \dfrac{5 \pm \sqrt{33}}{2}$
c) $x^2 = x + 6$ $x^2 - x - 6 = 0$ $x = \dfrac{-^-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)}$ $x = \dfrac{1 \pm \sqrt{25}}{2} = \dfrac{6}{2}$ or $\dfrac{-4}{2}$ $= 3$ or -2	d) $x^2 + 4x + 4 = 0$ $x = \dfrac{-4 \pm \sqrt{16 - 4(1)(4)}}{2(1)}$ $x = \dfrac{-4 \pm \sqrt{0}}{2}$ $x = -2$

Practice #8: Write an equation for a parabola that has x-intercepts of -4 and 6 with a y-intercept of 50.

$y = a(x+4)(x-6)$

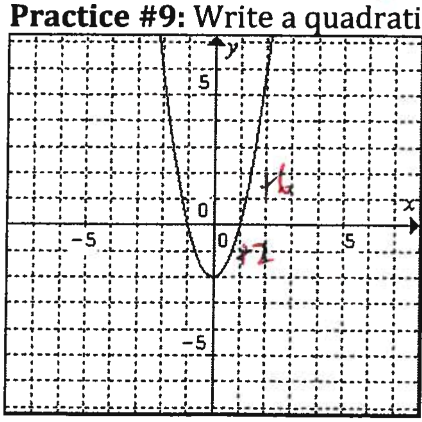
$50 = a(0+4)(0-6)$

$50 = a(-24)$

$a = 25/12$

$y = \frac{25}{12}(x+4)(x-6)$

Practice #9: Write a quadratic equation in factored form for this parabola:



$y = 2(x-1)(x+1)$

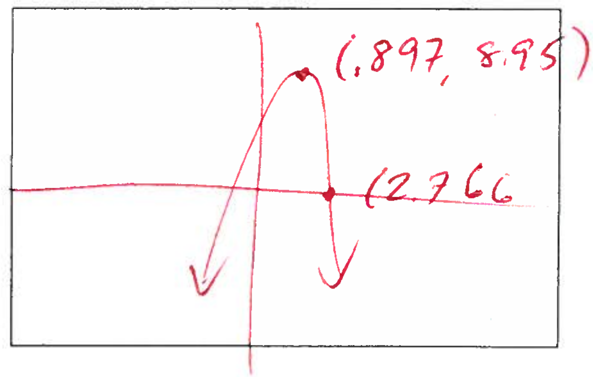
Practice #10: David dives from a spring board. His height, h metres, above the water t seconds after release is given by $h = -4.9t^2 + 8.8t + 5$

a) What is the domain and range for this word problem?

$t \geq 0, h \geq 0$

b) How high is the diving board?

5m (y-intercept)



c) What is David maximum height?

8.95m

d) How long is David in the air for?

2.8s

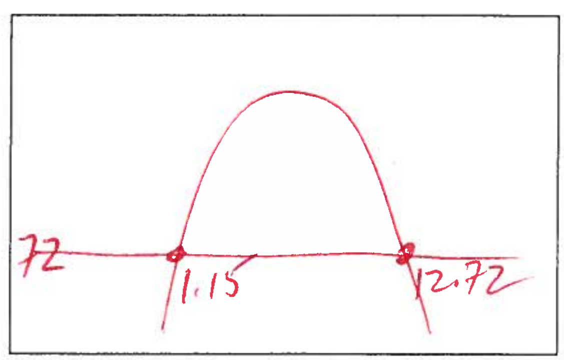
Practice #11: Bonnie launches a model rocket from the ground with an initial velocity of 68 m/s. The following function, $h(t)$, can be used to model the height of the rocket, in metres, over time, t, in seconds:

$$h(t) = -4.9t^2 + 68t$$

Bonnie's friend Sasha is watching from a lookout point at a safe distance. Sasha's eye level is 72m above the ground. At what time during the flight will the rocket be at Sasha's eye level?

$Y_1 = -4.9t^2 + 68t$

$Y_2 = 72$



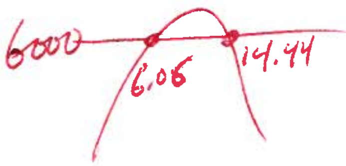
-10
20
-10
400

TWO ANSWERS

1.15s + 12.72s

Practice #11: A store rents an average of 750 video games each month at the current rate of \$4.50. The owners of the store want to raise the rental rate to increase the revenue to \$6000 per month. However, for every \$1 increase, they know that they will rent 30 fewer games each month because some people won't be willing to pay higher prices. What rate should they set for video game rentals?

$R = (750 - 30x)(4.50 + x)$



$4.50 + 6.06 = \$10.56$

OR

$4.50 + 14.44 = \$18.94$

Either would get revenue to \$6000

Practice #12: Johnny kicks a rugby ball in the air and it lands after 4 seconds. After 1 second the ball was 20 feet high. Write an equation to model the time vs. height of the ball.

$y = ax(x-4)$

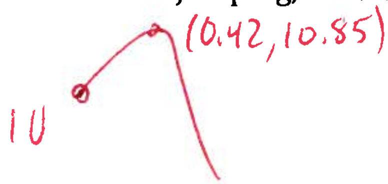
$20 = a(1)(1-4)$

$20 = -3a$

$20/{-3} = a$

$y = \frac{20}{-3}x(x-4)$

Practice #13: Gary is competing the National Diving Championships. He does a triple-back flip off the 10m tower. If Gary's maximum height of 10.85m occurred just 0.42s after jumping, then how long will it take him to hit the water?



$y = a(x-p)^2 + q$

$y = a(x - .42)^2 + 10.85$

$10 = a(0 - .42)^2 + 10.85$

$-.85 = .1764a$

$a = -4.82$

$y = -4.8(x - .42)^2 + 10.8$

GRAPH

zero

$= 2.34$ seconds

Practice #14: The length of a rectangular garden is 4m more than its width. Determine the dimensions of the garden if the area is 117 m².

$A = l \times w$

$117 = (w+4)w$

$Y_1 = 117$

$Y_2 = (w+4)(w)$

$w = 9$

$l = 9 + 4 = 13$

Dimensions $= 9 \times 13$