## Chapter 3+4: Trigonometry

### 3.4 Obtuse Angles

*Bigger than 90 degrees*

Fill in the chart below:

<table>
<thead>
<tr>
<th>Angle</th>
<th>Tangent</th>
<th>Sine</th>
<th>Cosine</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>0.577</td>
<td>0.5</td>
<td>0.866</td>
</tr>
<tr>
<td>45</td>
<td>1</td>
<td>0.707</td>
<td>1.707</td>
</tr>
<tr>
<td>60</td>
<td>1.73</td>
<td>0.866</td>
<td>0.5</td>
</tr>
<tr>
<td>90</td>
<td>error</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>120</td>
<td>-1.73</td>
<td>0.866</td>
<td>-0.5</td>
</tr>
<tr>
<td>135</td>
<td>-1</td>
<td>0.707</td>
<td>-1.707</td>
</tr>
<tr>
<td>150</td>
<td>-0.577</td>
<td>0.5</td>
<td>-0.866</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>210</td>
<td>0.577</td>
<td>-0.5</td>
<td>-0.866</td>
</tr>
<tr>
<td>225</td>
<td>1</td>
<td>-0.707</td>
<td>-1.707</td>
</tr>
<tr>
<td>240</td>
<td>1.73</td>
<td>-0.866</td>
<td>-0.5</td>
</tr>
<tr>
<td>270</td>
<td>error</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>300</td>
<td>-1.73</td>
<td>-0.866</td>
<td>0.5</td>
</tr>
<tr>
<td>315</td>
<td>-1</td>
<td>-0.707</td>
<td>1.707</td>
</tr>
<tr>
<td>330</td>
<td>-0.577</td>
<td>-0.5</td>
<td>0.866</td>
</tr>
<tr>
<td>360</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Observations:**

- *tangent repeats:* 0 → 180 = 180 → 360
- *sine = cosine (shifted 90)*
- *sine & cosine between -1 & 1*
- *sine 30 = cosine 60, etc.*
Sine and Cosine ratios are cyclical. Observe the graph of $y = \sin X$

Sine and Cosine ratios are based on the "unit circle":

\[
\begin{align*}
\sin 30^\circ &= 0.5 \\
\sin 150^\circ &= 0.5 \\
\cos 30^\circ &= \frac{0.866}{1} \\
\cos 150^\circ &= -\frac{0.866}{1}
\end{align*}
\]
The Problem:

\[ \sin X = 0.5 \quad \text{find } X \]

\[ \sin^{-1}(0.5) = 30^\circ \]

Problem: Calculator lies only gives smallest angle
\[ 180^\circ - 30^\circ = 150^\circ \]

Also has \( \sin \theta \) looks more like 150°

The Good News:

\[ \cos X = -0.866 \quad \text{find } X \]

\[ \cos^{-1}(-0.866) = 150^\circ \]

Simple Rule:

Between 0° and 180° sine has 2 solutions and cosine has 1 solutions.

This means to find a second answer for sine you always subtract your first solution from 180°.
**Examples:** Solve the following for angle $A$, where $0 \leq A \leq 180^\circ$.

1) $\sin A = 0.374$

\[
\sin^{-1}(0.374) = 22^\circ \\
\text{OR } 180 - 22 = 158^\circ
\]

2) $\sin A = 0.815$

\[
\sin^{-1}(0.815) = 54.6^\circ \\
\text{OR } 180 - 54.6 = 125.4^\circ
\]

3) $\cos A = 0.195$

\[
\cos^{-1}(0.195) = 78.8^\circ \\
\text{OR } 180 - 78.8 = 101.2^\circ \\
\text{COsINE ONLY HAS ONE ANSWER!}
\]

4) $\cos A = -0.943$

\[
\cos^{-1}(-0.943) = 161^\circ
\]

5) $\sin A = 0.513$

\[
\sin^{-1}(0.513) = 30.9^\circ \\
\text{OR } 180 - 30.9 = 149.1^\circ
\]
Assignment:

1. Each angles is inside a triangle (must be between 0 and 180). Which equations result in two difference answers?

   a) \( \sin A = 0.7071 \)  
   b) \( \cos A = -0.5 \)  
   c) \( \sin A = 0.9269 \)  
   d) \( \cos A = -0.7071 \)  
   
   e) \( \sin A = 0.8660 \)  
   f) \( \cos A = -1 \)  
   g) \( \sin A = \frac{3}{4} \)  
   h) \( \cos A = \frac{3}{4} \)  
   i) \( \cos A = -\frac{3}{4} \)

2. Given that \( 0^0 \leq \angle C \leq 180^0 \), determine the value(s) of \( \angle C \).

   a) \( \sin C = 0.9063 \)  
   b) \( \cos C = 0.5736 \)  
   c) \( \cos C = -0.7321 \)  
   d) \( \sin C = 0.4283 \)  
   
   e) \( \sin C = 0.5726 \)  
   f) \( \cos C = -0.3747 \)  
   
   g) \( \sin C = \frac{1}{2} \)  
   h) \( \cos C = \frac{1}{2} \)  
   i) \( \cos C = -\frac{1}{2} \)  
   j) \( \sin C = \frac{2}{3} \)  
   k) \( \sin C = \frac{1}{4} \)  
   
   l) \( \cos C = -\frac{5}{6} \)

Answers

1. a,c,e,g

2. a) 65°, 115°  
   b) 55°  
   c) 137°  
   d) 25°, 155°  
   e) 35°, 145°  
   f) 112°  
   g) 30°, 60°  
   h) 60°  
   i) 120°  
   j) 42°, 138°  
   k) 14°, 166°  
   l) 146°
Practice Quiz

1) Explain why \( \sin X = 0.5 \) has two possible answers between 0 and 180 degrees.

Two triangles in the unit circle have positive "y-values" of 0.5.

Sine function is cyclical.

2) Given that \( 0^\circ \leq \angle C \leq 180^\circ \), determine the value(s) of \( \angle C \) (to the nearest whole number)

a) \( \sin C = 0.866 \)

b) \( \cos C = 0.5 \)

c) \( \cos C = -0.866 \)

Practice Quiz Answer Key
1) see notes  2) a) 60, 120  b) 60  c) 150