Chapter 3: Trigonometry

3.3 Sine Law  For solving some types of non-right triangles

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Part 1 – Labeling a non-right triangle correctly

The Formula uses letters ABC like this:

But any three letters can be used
Assignment:
Complete the labels for each triangle. Make sure each pair of opposite sides/angles is matched with the same letter.

Label the following triangles with the given letters:

FUN

DIN

Create your own unique triangles and label them with the following letters

CAN

RIP
Part 2 – Where does sine law formula come from? Why does it work?

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

The sine law is a short cut to figure out this type of question without using SOH CAH TOA. The sine law is related to the height of a triangle.
Assignment

Follow the steps on the previous page of the notes to create equivalent ratios for these triangles:

\[ \sin M = \frac{h}{h} \quad \sin N = \frac{1}{m} \]
\[ n \sin M = h \quad m \sin N = h \]
\[ n \sin M = m \sin N \]
\[ \frac{h}{\sin N} = \frac{m}{\sin M} \]

\[ \sin X = \frac{1}{z} \quad \sin Z = \frac{1}{x} \]
\[ z \sin X = h \quad x \sin Z = h \]
\[ z \sin X = x \sin Z \]
\[ \frac{z}{\sin Z} = \frac{x}{\sin X} \]
Part 3 – How to set up the sine law ratio:

On the test and final exam, the sine law formula will be written like this:

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

But it can also be written upside-down:

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

And it can be made specific to the letters and numbers in the question:

Example:

\[
\frac{W}{\sin W} = \frac{h}{\sin H} = \frac{y}{\sin Y}
\]

Example:

\[
\frac{5}{\sin 90} = \frac{3}{\sin 37} = \frac{4}{\sin 53}
\]
Assignment:

1) Write the sine law ratio for each of the following triangles

   a) \[ \frac{\sin A}{a} = \frac{\sin R}{r} = \frac{\sin T}{t} \]

   b) \[ \frac{\sin 60}{\sin 40} > \frac{\sin 80}{\sin 40} \]

2) Write the sine law just for the two ratios given in the question:

   a) \[ \frac{x}{\sin X} = \frac{y}{\sin Y} \]

   b) \[ \frac{12}{\sin 65} = \frac{7}{\sin X} \]

   \[ \sin 65 = \sin X \]

   \[ \frac{12}{12} = \frac{\sin X}{7} \]
To use the sine law formula, you do not need all three ratios:

**Step #1** - Put the unknown value on the top of the first ratio and its opposite side/angle on the bottom

**Step #2** – Use the pair of opposites that are given in the question as the other ratio.

**Step #3** – Solve for the unknown

Example #1 – Solve for \( z \)

\[
\frac{z}{\sin 25} = \frac{x}{\sin X}
\]

\[
\frac{z}{\sin 88} = \frac{3}{\sin 25}
\]

\[
z = \frac{3 \sin 88}{\sin 25}
\]

\[
z = 7.1
\]

Example #2 – Solve for \( F \)

\[
\frac{\sin F}{29} = \frac{\sin 80}{67}
\]

\[
\sin F = \frac{29 \sin 80}{67}
\]

\[
\sin F = 0.42626
\]

\[
F = 25.2^\circ
\]
Assignment: \[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

1) Solve for the unknown side length or angle. Round your answer to one decimal place.

a) \[
\frac{x}{\sin 85} = \frac{6.5}{\sin 25} \\
\]
\[
x = \frac{6.5 \sin 85}{\sin 25} \\
x = 15.3
\]

b) \[
\frac{\sin X}{9.3} = \frac{\sin 47}{12.2} \\
\]
\[
\sin X = 9.3 \sin 47 \\
X = 33.9^\circ
\]

c) \[
\frac{\sin X}{8.4} = \frac{\sin 88}{9.7} \\
\]
\[
\sin X = 8.4 \sin 88 \\
X = 59.9^\circ
\]

BONUS d) \[
\frac{x}{\sin 15} = \frac{4.3}{\sin 37} \\
\]
\[
x = \frac{4.3 \sin 15}{\sin 37} \\
x = 6.5
\]

Answers: a) 15.3  b) 33.9  c) 59.9  d) 6.5
2) Sketch a triangle that corresponds to each equation below. Then solve for the unknown side length or angle. Round your answer to one decimal place.

\[ a) \quad \frac{x}{\sin50^\circ} = \frac{8.0}{\sin60^\circ} \]

\[ x = \frac{8.0 \cdot \sin50^\circ}{\sin60^\circ} \]
\[ x = 7.1 \]

\[ b) \quad \frac{\sin X}{6.0} = \frac{\sin72^\circ}{10.0} \]

\[ \frac{\sin X}{6.0} = \frac{0.5706339}{10.0} \]
\[ \sin X = 34.8^\circ \]

\[ c) \quad \frac{n}{\sin43^\circ} = \frac{9.5}{\sin85^\circ} \]

\[ h = \frac{9.5 \cdot \sin43^\circ}{\sin85^\circ} \]
\[ h = 6.5 \]

\[ BONUS \quad d) \quad \frac{12.5}{\sin N} = \frac{14.0}{\sin88^\circ} \]

\[ \sin N = \frac{12.5}{14.0} \]
\[ \sin N = 0.8 \]
\[ N = 63.2^\circ \]

Answers:

a) 7.1  
 b) 34.8  
 c) 6.5  
 d) 63.2
Practice Quiz

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

1) Using basic trig ratios and the height of the triangle, write the steps to prove the Sine Law for the following triangle

\[ \sin D = \frac{h}{f} \]
\[ \sin F = \frac{h}{g} \]
\[ f \sin D = h \]
\[ g \sin F = h \]
\[ \frac{f \sin D}{g \sin F} = \frac{h}{h} \]
\[ \frac{f \sin D}{g \sin F} = \frac{\sin D}{\sin F} \]

2) Solve for the unknown side length or angle. Round your answer to one decimal place.

\[ \frac{x}{\sin 32^\circ} = \frac{13}{\sin 78^\circ} \]
\[ x = \frac{13 \sin 32^\circ}{\sin 78^\circ} \]
\[ x = 7.0 \]

Answers
1) see notes “Part 2” and change ABC to DEF
2) 7.0