Chapter 3: Trigonometry

3.10 Sine or Cosine?

Sine Law
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Cosine Law
\[
c^2 = a^2 + b^2 - 2ab \cos C
\]

ASA or AAS

SAS

ASS

SSS
Example #1:

Example #2:
1) Determine whether the Law of Sines or the Law of Cosines would be used to begin the solution process for each triangle.

a) \[ \triangle ABC \]
   - \( A = 12 \)
   - \( B = 10 \)
   - \( C = 15 \)
   - Law of \( \cos \)

b) \[ \triangle ABC \]
   - \( A = 12 \)
   - \( B = 10 \)
   - \( C = 40° \)
   - Law of \( \sin \)

c) \[ \triangle ABC \]
   - \( A = 25° \)
   - \( B = 50° \)
   - \( C = 103° \)
   - Law of \( \cos \)

d) \[ \triangle ABC \]
   - \( A = 61° \)
   - \( B = 27° \)
   - \( C = 10 \)
   - Law of \( \sin \)

e) \[ \triangle ABC \]
   - \( A = 70° \)
   - \( B = 12 \)
   - \( C = 8 \)
   - Law of \( \sin \)

f) \[ \triangle ABC \]
   - \( A = 120° \)
   - \( B = 120° \)
   - \( C = 20° \)
   - Law of \( \cos \)

2) Given the indicated parts of \( \triangle ABC \), what angle or side should be found first, and which formula should be used to find it?

a) \[ \triangle ABC \]
   - \( a = 6 \)
   - \( b = 9 \)
   - \( c = 11 \)
   - Law of \( \cos \)

b) \[ \triangle ABC \]
   - \( a = 8 \)
   - \( b = 10 \)
   - \( c = 12 \)
   - Law of \( \sin \)

c) \[ \triangle ABC \]
   - \( a = 7 \)
   - \( b = 9 \)
   - \( c = 11 \)
   - Law of \( \cos \)

...
3) Solve each triangle ABC. Round answers to one decimal place:

   a) $\angle A = 50^\circ, b = 10, c = 15$

   b) $\angle B = 36^\circ, a = 4, c = 10$

   c) $\angle C = 60^\circ, b = 4, a = 8$

   d) $a = 2, b = 3, c = 4$
e) \( a = 9, b = 14, c = 11 \)

f) \( b = 4, c = 1, \angle A = 120^\circ \)

g) \( \angle A = 28^\circ, \angle B = 42^\circ, c = 18.2 \)

h) \( \angle B = 63^\circ, b = 8, c = 10 \)
i) $\angle B = 41^\circ, a = 11, c = 6$

**Answer Key**

1a) Cosine Law   b) Sine Law   c) Neither   d) Sine Law   e) Cosine Law   f) Sine Law


e) Angles add to 180: angle $A$   f) Can’t be solved   g) Sine Law: side $b$   h) Cosine Law: side $c$

3) a) $\angle B = 41.8^\circ, \angle C = 88.2^\circ, a = 11.5$  
   b) $\angle B = 19.2^\circ, \angle C = 124.8^\circ, b = 7.2$

c) $\angle A = 90^\circ, \angle B = 30^\circ, a = 6.9$   d) $\angle A = 29.0^\circ, \angle B = 46.6^\circ, \angle C = 104.4^\circ$

e) $\angle A = 40.0^\circ, \angle B = 88.2^\circ, \angle C = 51.8^\circ$   f) $\angle B = 49.1^\circ, \angle B = 10.9^\circ, a = 4.6$

   g) $\angle C = 110^\circ, a = 9.1, b = 13.0$   h) No Solution   i) $\angle A = 107.7^\circ, \angle C = 31.3^\circ, b = 7.6$
**Sine or Cosine Word Problems**

Basic Trig Ratios

**SOH CAH TOA**

Geometry Rules

**Angles in a triangle = 180**

Sine Law

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Law

$a^2 = b^2 + c^2 - 2bc \cos A$

**Example #1**

Two security cameras in an expensive jewelry store must be adjusted to monitor an expensive diamond. The cameras are mounted 12 ft. above the floor, directly across from each other on opposite walls. The walls are 24 ft. apart. The diamond is displayed in a case 4 ft. high. The distance from the camera on the left to the diamond is 11 ft. Both cameras must aim directly at the diamond. What is the angle of depression for both cameras?
Example #2
Mr. Seywerd is in a Search and Rescue helicopter flying 500m over Cultus Lake. When he looks directly SE, he sees a person in distress at the bottom of a cliff. The angle of depression to the person is 40 degrees. When he looks back directly NW, he sees a firetruck approaching. The angle of depression to the firetruck is 20 degrees. How far away is the firetruck from the person in distress? If the firetruck approaches with a directional average speed of 70 km/h, how long will it take for the firetruck to reach the scene of the accident?
Assignment:
1) A kayak leaves Rankin Inlet, Nunavut, and heads due east for 5.0 km, as shown in the diagram. At the same time, a second kayak travels in a direction S60E from the inlet for 4.0 km. How far apart, to the nearest tenth of a kilometer, are the kayaks?

2) A crane stands on top of a building, as shown. How far is the point on the ground from the base of the building, to the nearest tenth of a metre? How tall is the crane?

3) A tree is growing on a hillside, as shown. The hillside is inclined at an angle of 15° to the horizontal. The tree casts a shadow uphill. How tall is the tree, to the nearest metre?
4) A radar operator on a ship discovers a large sunken vessel lying parallel to the ocean surface, 200m directly below the ship. The length of the vessel is a clue to which wreck has been found. The radar operator measures the angles of depression to the front and back of the sunken vessel to be 56 and 62. How long, to the nearest tenth of a metre, is the sunken vessel?

5) Fred and Agnes are 520m apart. As Brendan flies overhead in an airplane, they estimate the angle of elevation of the airplane. Fred, looking South, estimates the angle of elevation to be 60. Agnes, looking north, estimates it to be 40. What is the altitude of the airplane, to the nearest tenth of a metre?

**Answers**

1) 2.5 km  
2) 43.2 m, 13.3 m  
3) 8 m  
4) 241.2 m  
5) 293.9 m  
6) 879.3 m, 40 seconds
**Practice Quiz**
Solve each triangle ABC. Round answers to one decimal place:

1) $\angle A = 126^\circ, b = 9, a = 12.2$

2) $a = 6, b = 7, c = 12$

3) Ryan is in a police helicopter, 400m directly above the Sea to Sky highway near Whistler. When he looks north, the angle of depression to a car accident is 65. When he looks south, the angle of depression to the approaching ambulance is 30. How far away from the accident is the ambulance? How long will it take to get to the accident?

**Answers:**
1) $\angle B = 22.6^\circ, \angle C = 31.4^\circ, a = 18.9$
2) $\angle A = 20.8^\circ, \angle B = 24.5^\circ, \angle C = 134.7^\circ$
3) 157.0 km