7.1 Patterns and Predictions

Patterns are widely used in mathematics to reach __________________________

Remember the patterns that you find may be, but are __________________________

Famous Math Patterns

1) The FIBONACCI Sequence

1, 1, 2, 3, 5, 8, 13, ___, ___, ____

The GOLDEN RATIO is...

2) PASCAL’S Triangle
**Example 1:** Find the patterns.

a) \( 1 \times 1 = 1 \)

\[ 11 \times 11 = 121 \]

\[ 111 \times 111 = 12321 \]

\[ 1111 \times 1111 = \_\_\_\_\_ \]

\[ 11111 \times 11111 = \_\_\_\_\_\_\_ \]

b) \( 1 \times 9 + 2 = 11 \)

\[ 12 \times 9 + 3 = 111 \]

\[ 123 \times 9 + 4 = \_\_\_\_\_ \]

\[ 1234 \times 9 + 5 = \_\_\_\_\_\_\_ \]

**Example 2:** Draw the next shape in the pattern, and predict the number of pieces in the next two patterns.

---

3 pieces

9 pieces

18 pieces
Example 3: Find the next two numbers in the following patterns.

a) 3, 6, 8, 16, 18, 36, 38, ___, ___

b) 2, 4, 5, 10, 12, 24, 27, ___, ___

Example 4: Which number(s) in the last group are zuts?
Assignment

1. Study the pattern and predict the missing values
   
   a) \[ 9 \times 9 + 7 = 88 \]
      
      \[ 98 \times 9 + 6 = 888 \]
      
      \[ 987 \times 9 + 5 = _______ \]
      
      \[ 9876 \times 9 + 4 = _______ \]
      
      \[ 98765 \times 9 + 3 = _______ \]
      
   b) \[ 9^2 = 81 \]
      
      \[ 99^2 = 9801 \]
      
      \[ 999^2 = 998001 \]
      
      \[ 9999^2 = _______ \]
      
      \[ 99999^2 = _______ \]
      
   c) \[ 1^2 + 1 + 2 = 4 \]
      
      \[ 2^2 + 2 + 3 = 9 \]
      
      \[ 3^2 + 3 + 4 = 16 \]
      
      \[ 4^2 + 4 + 5 = _______ \]
      
      \[ 5^2 + 5 + 6 = _______ \]
      
   d) \[ 1 = 1 \]
      
      \[ 1 + 2 = 3 \]
      
      \[ 1 + 2 + 3 = 6 \]
      
      \[ 1 + 2 + 3 + 4 = 10 \]
      
      \[ 1 + 2 + 3 + \cdots + 10 = _______ \]
      
   e) \[ 1 = 1 \]
      
      \[ 1 + 3 = 4 \]
      
      \[ 1 + 3 + 5 = 9 \]
      
      \[ 1 + 3 + 5 + \cdots + 15 = _______ \]
      
   f) \[ 2 = 2 \]
      
      \[ 2 + 4 = 6 \]
      
      \[ 2 + 4 + 6 = 12 \]
      
      \[ 2 + 4 + 6 + 8 = _______ \]
      
      \[ 2 + 4 + \cdots + 16 = _______ \]
      
   g) \[ 1 = 1 \]
      
      \[ 1 + 3 = 4 \]
      
      \[ 1 + 3 + 5 = 9 \]
      
      \[ 1 + 3 + 5 + 7 = 16 \]
      
      \[ 1 + 3 + 5 + \cdots + 59 = _______ \]
      
   h) \[ 2 = 2 \]
      
      \[ 2 + 4 = 6 \]
      
      \[ 2 + 4 + 6 = 12 \]
      
      \[ 2 + 4 + 6 + 8 = 20 \]
      
      \[ 2 + 4 + 6 + \cdots + 60 = _______ \]
      
   i) \[ 1 + 9 \times 0 = 1 \]
      
      \[ 2 + 9 \times 1 = 11 \]
      
      \[ 3 + 9 \times 12 = _______ \]
      
      \[ 4 + 9 \times 123 = _______ \]
      
      \[ \underline{11111} = 11111 \]
      
   j) \[ 8 + 9 \times 0 = 8 \]
      
      \[ 7 + 9 \times 9 = 88 \]
      
      \[ 6 + 9 \times 98 = _______ \]
      
      \[ 5 + 9 \times 987 = _______ \]
      
      \[ \underline{\quad} = \underline{\quad} \]
2. Study the pattern, and predict the next two terms.
   a) 2, 3, 5, 8, 12, _____, _____
   b) 20, 25, 31, 38, 46, _____, _____
   c) 10, 7, 12, 9, 14, _____, _____
   d) 3, 6, 11, 18, 27, 38, _____, _____
   e) 2, 6, 15, 31, 56, _____, _____
   f) 2, 6, 12, 20, 30, _____, _____
   g) 15, 19, 25, 33, 43, _____, _____
   h) 1, 2, 5, 14, 41, _____, _____
   i) 3, 5, 11, 29, 83, _____, _____
   j) 59, 52, 55, 48, 51, 44, 47, _____, _____

3. What pattern is observed in the following? (Hint: think about odd and even numbers.)
   a) \[5 + 7 = 12\] , \[47 + 31 = 78\]
      \[-9 + 3 = 6\] , \[(-17) + (-41) = -58\]
     
   b) \[4 + 12 = 16\] , \[42 + 16 = 58\]
      \[-8 + 4 = -4\] , \[(-12) + (-8) = -20\]
     
   c) \[6 + 7 = 13\] , \[14 + (-17) = -3\]
      \[-13 + 4 = -9\] , \[(-4) + (-7) = -11\]
     
   d) \[3 \times 6 = 18\] , \[7 \times 8 = 56\]
      \[-5 \times 4 = -20\] , \[(-9) \times (-4) = 36\]
     
   e) \[3 \times 5 = 15\] , \[-5 \times 11 = -55\]
      \[-7 \times 9 = -63\] , \[(-1) \times (-13) = 13\]
     
   f) \[\frac{18}{3} = 6\] , \[\frac{12}{3} = 4\]
      \[\frac{20}{5} - 4\] , \[\frac{30}{3} = 10\]
4. Determine the number of matchsticks used in the 100th pattern.
   a) 
   b) 
   c) 
   d) 

KEY

1. a) 8888, 8888, 888888   b) 99 980 001, 9999 800 001   c) 25, 36    d) 55
   e) 64   f) 20, 72   g) 30^2 = 900   h) 30 × 31 = 930   i) 111, 1111, 5 + 9 × 1234
   j) 888, 8888, 4 + 9 × 9876 = 8888

2. a) 17, 23    b) 55, 65    c) 11, 16    d) 51, 66    e) 92, 141    f) 42, 56
   g) 55, 69    h) 122, 365    i) 245, 731    j) 40, 43

3. a) An odd number plus an odd number equals an even number.
    b) An even number plus an even number equals an even number.
    c) An odd number plus an even number equals an odd number.
    d) An odd number times an even number equals an even number.
    e) An odd number times an odd number equals an odd number.
    f) An even number divided by an odd number is even.

4. a) 4 + 3(99) = 301    b) 6 + 5(99) = 501    c) 5 + 4(99) = 401    d) 7 + 6(99) = 601
1.1 Patterns and Predictions

Patterns are widely used in mathematics to reach logical conclusions. Remember the patterns that you find may be, but are not necessarily true.

Famous Math Patterns

1) The FIBONACCI Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55

The GOLDEN RATIO is the last number in the Fibonacci sequence divided by the previous number.